Some phenomena in the Universe evolution to present Inert Doublet stage

I. F. Ginzburg

Sobolev Inst. of Mathematics, SB RAS

and Novosibirsk State University

Novosibirsk, Russia

Inert Doublet Model. Brief review

SM with standard Higgs field ϕ_S is supplemented by Higgs field ϕ_D, having no interaction with matter fields and v.e.v. $=0$).
G. Despande, L. Ma *Phys.Rev.* **D18** (1978) 2574; many papers now

Lagrangian: $$\mathcal{L}=\mathcal{L}_{gf}^{SM}+\mathcal{L}_Y+\frac{1}{2}(D_\mu\phi_S D_\mu\phi_S^\dagger+D_\mu\phi_D D_\mu\phi_D^\dagger)-V\,.$$

$\mathcal{L}_{gf}^{SM}$: $SU(2)\times U(1)$ SM interaction of gauge bosons and fermions;
$\mathcal{L}_Y$: Yukawa interaction of fermions with Higgs field ϕ_S only.

V: Higgs potential, forbidding (ϕ_S, ϕ_D) mixing:

$$V = -\frac{1}{2}\left(m_{11}^2(\phi_S^\dagger\phi_S)+m_{22}^2(\phi_D^\dagger\phi_D)\right) + \\ +\frac{1}{2}\left(\lambda_1(\phi_S^\dagger\phi_S)^2+\lambda_2(\phi_D^\dagger\phi_D)^2\right) + \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D)+ \\ +\lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S) + \frac{\lambda_5}{2}\left((\phi_S^\dagger\phi_D)^2+(\phi_D^\dagger\phi_S)^2\right) + V_0.$$

We fix $\lambda_5 < 0$, real without loss of generality.

Potential is positive at large quasi-classical values for fields ϕ_i if only

$$\lambda_1 > 0\,, \quad \lambda_2 > 0, \quad R+1 > 0; \quad R = \lambda_{345}/\sqrt{\lambda_1\lambda_2}, \quad \lambda_{345} = \lambda_3+\lambda_4+\lambda_5.$$

To describe IDM, parameters of potential should lie in some bounded area of parameters m_{ii}^2, λ_i.

Useful abbreviations: $\mu_1 = \dfrac{m_{11}^2}{\sqrt{\lambda_1}}, \qquad \mu_2 = \dfrac{m_{22}^2}{\sqrt{\lambda_2}}$

Temperature dependence

At the finite temperature the ground state of system is given by minimum of the Gibbs potential $V_G = Tr\left(Ve^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right)$. At high enough temperatures in the main approximation V_G has the same form as V with the same λ_i, and mass terms evolving with temperature

$$m_{11}^2(T) = m_{11}^2(0) - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2(0) - c_2 T^2\,,$$
$$c_1 = (3\lambda_1 + 2\lambda_3 + \lambda_4)/12 + (3g^2 + g'^2)/32 + (g_t^2 + g_b^2)A,$$
$$c_2 = (3\lambda_2 + 2\lambda_3 + \lambda_4)/12 + (3g^2 + g'^2)/32\,.$$

g and g' are coupling constants of gauge EW interaction. The Yukawa coupling constants of SM for t and b quarks are $g_t \approx 1$ and $g_b \approx 0.03$. Simple analysis shows that in the case of neutral DM particle

At $R > 0$ $\quad c_1 > 0,\ c_2 > 0\,,$

At $R < 0$ $\quad$ *both signs of* $c_1 > 0,\ c_2$ *are possible, but* $c_1 + c_2 > 0\,.$

Extrema of potential

The extrema of the potential define the values $\langle\phi_{S,D}\rangle$ of the fields $\phi_{S,D}$ via equations: $\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle} = 0$. For each extremum with $\langle\phi_S\rangle \neq 0$ we choose the z axis in the weak isospin space so that $\langle\phi_S\rangle = \begin{pmatrix} 0 \\ v_S \end{pmatrix}$ with real $v_S > 0$ ("neutral direction"). After this choice the most general form extremum is $\langle\phi_S\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle\phi_D\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_D \end{pmatrix}$.

The vacuum with $u \neq 0$ is excluded in the model.

Complete set of these solutions contains 1 electroweak symmetry preserving extremum EWs and 3 electroweak symmetry violating (*EWv*) extrema: inert extremum I_1, inert-like extremum I_2 and mixed extremum M. We list their v.e.v.'s and extrema energies $\bar{\mathcal{E}}_a = \mathcal{E}_a - V_0$:

$$\boldsymbol{EWs}: v_D = 0, \quad v_S = 0, \qquad \bar{\mathcal{E}}_{EWs} = 0;$$

$$\boldsymbol{I_1}: v_D = 0,\ v_S^2 = \frac{m_{11}^2}{\lambda_1}, \quad \bar{\mathcal{E}}_{I_1} = -\frac{\mu_1^2}{8};$$

$$\boldsymbol{I_2}: v_S = 0,\ v_D^2 = \frac{m_{22}^2}{\lambda_2}, \quad \bar{\mathcal{E}}_{I_2} = -\frac{\mu_2^2}{8};$$

$$\boldsymbol{M}: v_S^2 = \frac{\mu_1 - R\mu_2}{\sqrt{\lambda_1}(1-R^2)}, \quad v_D^2 = \frac{\mu_2 - R\mu_1}{\sqrt{\lambda_2}(1-R^2)}; \quad \bar{\mathcal{E}}_M = -\frac{\mu_1^2 + \mu_2^2 - 2R\mu_1\mu_2}{8(1-R^2)}.$$

If some v_a^2, given by these equations, are negative, corresponding extremum is absent.

We assume that our world is described by I_1 (inert phas

In this state we have ($G^{\pm}$, G – Goldstone modes.)

$$\phi_S = \begin{pmatrix} G^{+} \\ \dfrac{v+h+iG}{\sqrt{2}} \end{pmatrix}, \quad \phi_D = \begin{pmatrix} D^{+} \\ \dfrac{D_H+iD_A}{\sqrt{2}} \end{pmatrix}.$$

For the to-day IDM state $v = 246\ GeV$.

We denote by M_h, M_D, M_A, $M_{\pm} \equiv M_{+}$ masses of h, D_H, D_A and $D^{\pm}$. Scalar h interact with the fermions and gauge bosons just as Higgs boson in the SM. As in SM, $M_h^2 = \lambda_1 v^2,$. D-scalars D, D_A, $D^{\pm}$ don't couple to fermions. The lightest from these D-scalars can play a role of DM particle, at $\lambda_4 + \lambda_5 < 0$ it is neutral:

$$M_D^2 = \sqrt{\lambda_2}\frac{R\mu_1 - \mu_2}{2}, \;\; M_A^2 = M_D^2 - v^2\lambda_5, \;\; M_{\pm}^2 = M_D^2 - v^2\frac{\lambda_4+\lambda_5}{2}.$$

This state can be ground state (vacuum) if only

$$m_{11}^2 > 0 \;\; at \;\; any \;\; R, \quad \mu_1 > \mu_2 \;\; at \;\; R > 1, \quad R\mu_1 > \mu_2 \;\; at \;\; |R| < 1.$$

Inert-like phase I_2 looks similar to the inert phase.

ϕ_D **looks** similar to Higgs field in SM. Its 4 components are splitted into 3 Goldstone modes + observable Higgs boson D_H with mass $M^2_{D_h} = \lambda_2 v^2$ and $M^2_{D_S} = \sqrt{\lambda_1}\dfrac{R\mu_2 - \mu_1}{2}$. D_h has no coupling to fermions – fermions are massless.

If this state is vacuum, we see no candidates for DM particle.

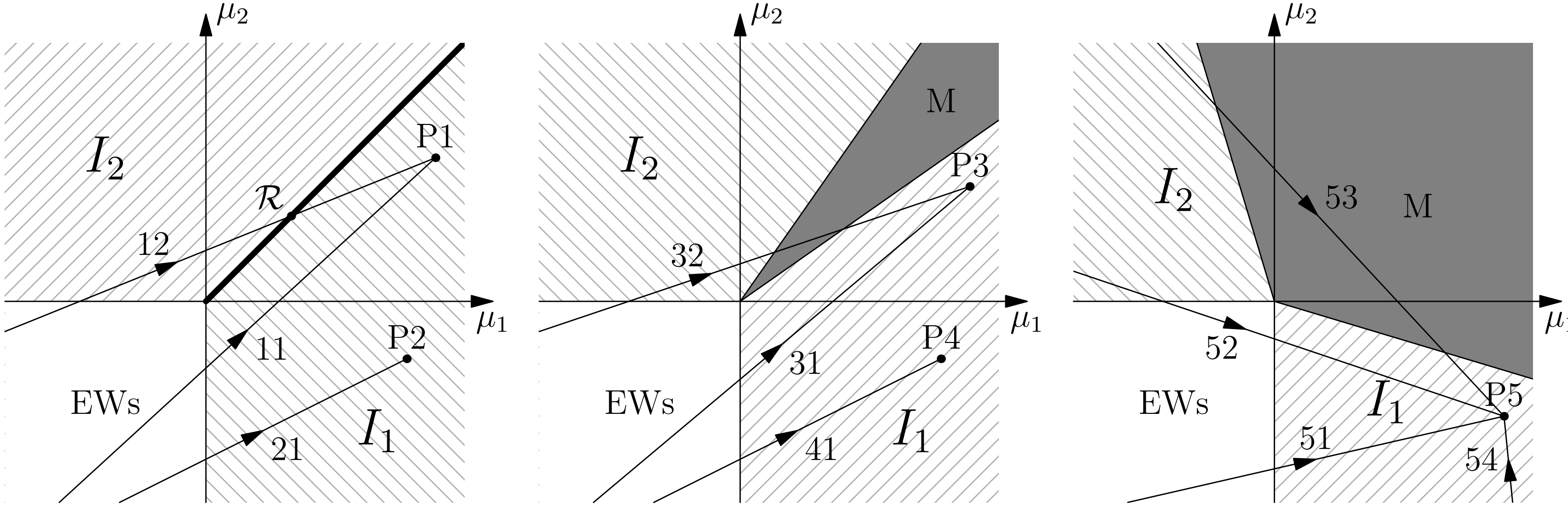
Certainly, at $m^2_{22} < 0$ this I_2 does not exist.

M (mixed phase): This phase is similar to that in 2HDM with Model I for Yukawa interaction. Scalars h, H, A, $H^{\pm}$ + 3 Goldstones mix components of ϕ_D and ϕ_S.

This extremum can be minimum if only $R^2 < 1$.

Thermal evolution of Universe

During cooling of the Universe mass terms in V were changed $\Rightarrow$ phase states were changed. Possible ways of evolution are shown here:

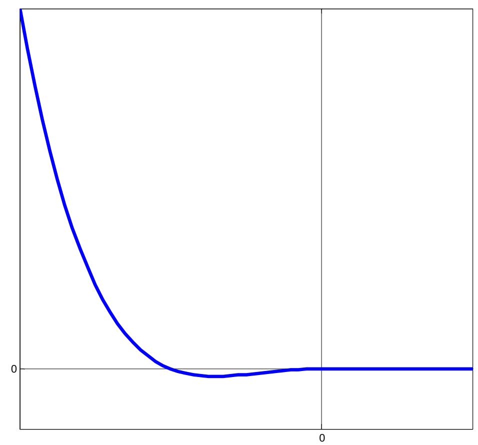


. $R > 1$ $1 > R > 0$ $0 > R > -1$

New: Limitation for thermal coefficient c_1

In the modern inert phase total vacuum energy density is the sum of energy of matter $A\sigma T^4$ and calculated vacuum energy $\bar{\mathcal{E}}_{I1} + V_0$
$= -\left(m_{11}^4(T) - m_{11}^4(0)\right)/(8\lambda_1) \equiv -\left([m_{11}^2(0) - c_1 T^2]^2 - m_{11}^4(0)\right)/(8\lambda_1)$.
At $c_1 > 0$ this sum increases with growth of temperature. The state with $T = 0$ has lowest energy.

At $c_1 < 0$ this sum decreases with growth of temperature from 0. It has minimum at some $T = T_m \neq 0$. $\Rightarrow$ Cooling down of Universe must be stopped at $T = T_m$!
Therefore, the case $c_1 < 0$ is excluded if IDM describes our world.



New: Mixed phase (Ray 32)

In the considered model the extremum equations for v.e.v.'s give not v_D, but v_D^2 (sign of v_S is fixed). Therefore, mixed extremum is degenerated in the sign of v_D, there are 2 mixed vacua, with positive and negative v_D.

They can be distinguished by the value of couplings of h and H to gauge bosons $\propto \cos(\beta \pm \alpha)$, etc.

Mixed phase is made of domains with $v_D = \pm|v_D|$.
The height of domain wall is given by position of lowest saddle extremum among considered above.

At $\mu_1 > \mu_2$ it is inert extremum I_1 with $E_b = \mathcal{E}_{I1} - \mathcal{E}_M = \dfrac{(\mu_1 R - \mu_2)^2}{8(1-R^2)}$.
Note that inert extremum become saddle point when $M_D^2 \propto (\mu_1 R - \mu_2)$ become negative. Near the second order transition point $T_{M,I1}$ we have $(\mu_1 R - \mu_2) = A_1(T_{M,I1}^2 - T^2)$ with $A_1 > 0$.

At $\mu_2 > \mu_1$ lowest saddle extremum become inert-like I_2 with $E_b = \mathcal{E}_{I2} - \mathcal{E}_M = \dfrac{(\mu_2 R - \mu_1)^2}{8(1-R^2)}$. Near the second order transition point $T_{M,I2}$ we have $(\mu_2 R - \mu_1) = A_2(T^2 - T_{M,I2}^2)$ with $A_2 > 0$.

Evolution through mixed phase (Ray 32)

Starting from EWs state the Universe comes to the Inert-like phase I_2 having no DM particles (second order phase transition).
At cooling down to temperature $T = T_{M,I2}$ the Universe come to the mixed phase. As usual near the phase transition system has huge fluctuation. However, in contrast with standard picture when fluctuations contains islands of old and new phase, in this case we will have islands of I_2 phase and mixed phase of two types, with positive and negative v_D. Map of these islands is constantly changing. The characteristic correlation radius is $R_c(T) \propto 1/\sqrt{|\mu_2 R - \mu_1|} \propto 1/\sqrt{|T^2 - T_{M,I2}^2|}$.
Near the transition the sphere of radius R_c contains huge number of fluctuations.

With the growth of temperature (at $\mu_2 > \mu_1$)) correlation radius decreases, domains become smaller than R_c, they become bubbles with opposite sign of v_D in each of them and with surface tension $\sigma_s \sim E_b R_c$. The curved surface of this bubble is under pressure $\sim \sigma_s / r$, where r is the local radius of curvature. It results growth and disappearance of domains with effective heating of medium. At cooling down to the region $\mu_1 > \mu_2$ this process is continued with new values of E_b and R_c. At $T = T_{M,I1}$ the second order phase transition to the inert phase takes place.
It is not completely clear what is the structure of Universe before this transition. The local growth and disappearance of domains is fast process with velocity similar to the speed of light c. But in general this process is slow diffuse process. So that it is unclear whether Universe will be uniform or not in the beginning of modern inert phase.
Note that allowed values of parameters allow to have latter phase transition at low enough temperature.