Light inflaton – connecting cosmology with experiment

Based on: A.Anisimov, Y.Bartocci, FB, Phys. Lett. B **659**, 703 (2008) FB, D.Gorbunov, JHEP **05** (2010) 010

Fedor Bezrukov

Ludwig-Maximilians-Universität München ARNOLD SOMMERFELD CENTER for theoretical physics Germany



SCALARS 2011 Warsaw, Poland August 26–29, 2011

Outline



Our present knowledge of particle physics and the Universe

- Standard Model
- SM problems in laboratory and in cosmology
- Minimal extension approach
- Inflating with a light inflaton
 - Inflationary model
 - Bounds from cosmology inflation and reheating
 - Experimental detection of the inflaton
 - Higgs mass bounds

3 Summary



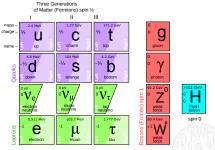
Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Standard Model – describes nearly everything that we know

Gauge theory $SU(3) \times SU(2) \times U(I)$ Describes (together with Einstein gravity)

- all laboratory experiments

 electromagnetism,
 nuclear processes, etc.
- all processes in the evolution of the Universe after the Big Bang Nucleosynthesis (T < 1 MeV, t > 1 sec)



Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Standard Model has experimental problems

- Laboratory
 - Neutrino oscillations
- Cosmology
 - Baryon asymmetry of the Universe
 - Dark Matter
 - Dark Energy
 - Inflation
 - Horizon problem (and flatness, entropy, ...)
 - Initial density perturbations



Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Neutrino oscillations



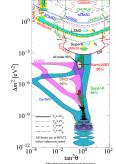
SAGE neutrino observatory (solar oscillations evidence $v_e \rightarrow v_u$)

SuperKamiokande (atmospheric oscillations $v_{\mu} \rightarrow v_{\tau}$)

Reactor neutrinos, accelerator neutrinos

Oscillation parameters

Δm_{21}^2	$7.59{\scriptstyle \pm 0.20} \times 10^{-5} \; eV^2$
$\sin^2 2\theta_{12}$	0.87 ± 0.03
$ \Delta m_{32}^2 $	$2.43{\pm}0.13{ imes}10^{-3}~eV^2$
$\sin^2 2\theta_{23}$	> 0.92
$sin^2 2\theta_{13}$	< 0.15





August 26-29, 2011

Light inflation - cosmology and experiment

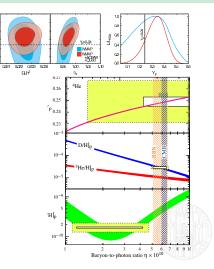
Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Baryon asymmetry of the Universe

- Current universe contains baryons and no antibarions
- Current baryon density

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq 6.1 \times 10^{-10}$$

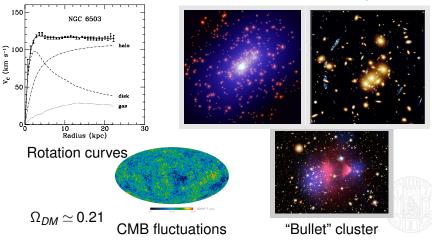
 Does not fit into the SM (too weak CP violation, too smooth phase transition)



Aburdance,

Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Dark Matter



Gravitational lensing

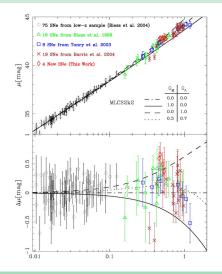
August 26-29, 2011

Fedor Bezrukov

Light inflation - cosmology and experiment

Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Dark Energy



← Supernova type la redshifts

accelerated expansion of the Universe today $\Omega_{\Lambda}\simeq 0.74$

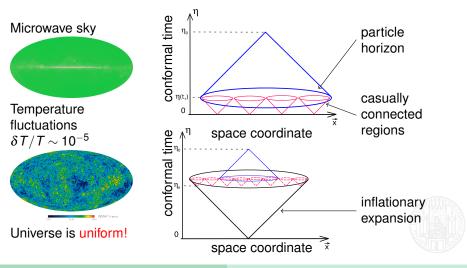
Different from inflation

- Much lower scale
- No need to stop it

Can be explained "just" by a cosmological constant

Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Inflation evidence - horizon problem

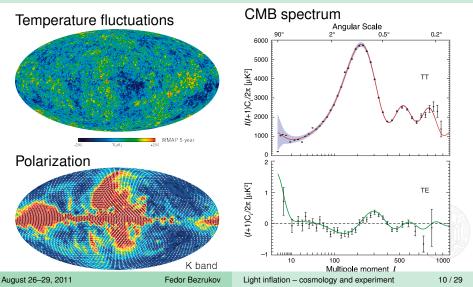


August 26-29, 2011

Fedor Bezrukov

Standard Model SM problems in laboratory and in cosmology Minimal extension approach

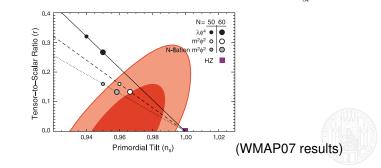
CMB gives measured predictions from inflation



Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Inflationary parameters from CMB

- Spectrum of primordial scalar density perturbations is just a bit not flat $n_s 1 \equiv \frac{d \log \mathscr{P}_{\mathscr{R}}}{d \log k}$
- Tensor perturbations are compatible with zero $r \equiv \frac{\mathscr{P}_{grav}}{\mathscr{P}_{a}}$



Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Let us expand the model in a minimal way

I will follow a "Minimal" approach

Explain the experimental facts with

- minimal number of new particles
- no new physical scales

Different situation in usual approaches

Solve hierarchy problems first

• Supersymmetry, Extra dimensions New physics at TeV energies – "masks" us from early Universe

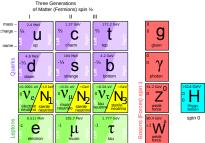
Standard Model SM problems in laboratory and in cosmology Minimal extension approach

vMSM — describes all, except for inflation

SM symmetrically extended by right handed (Majorana) neutrinos N_i

Describes

- DM by keV scale neutrino *N*₁
- BAU via leptogenesys by two heavier (GeV scale) neutrinos N_{2,3}



However - nothing about scalars - not to be told here

[Asaka, Blanchet, Shaposhnikov'05, Asaka, Shaposhnikov'05]

Standard Model SM problems in laboratory and in cosmology Minimal extension approach

Examples of minimal extensions leading to inflation

vMSM (for DM and Baryogenesys) +

• Inflation with light inflaton

[Shaposhnikov, Tkachev'06] [Anisimov, Bartocci, FB'08] [FB, Gorbunov'09]

(Introduces new particle)

Higgs boson inflation
 [FB, Shaposhnikov'08]
 [FB, Gorbunov, Shaposhnikv'08]

[FB, Magnin, Shapshnikov'08]

(Modifies Higgs-gravity interaction, new scales M_P/ξ ,

 $M_P/\sqrt{\xi}$ • R^2 (scalaron) inflation

[Starobinsky'80] [Gorbunov, Panin'10]

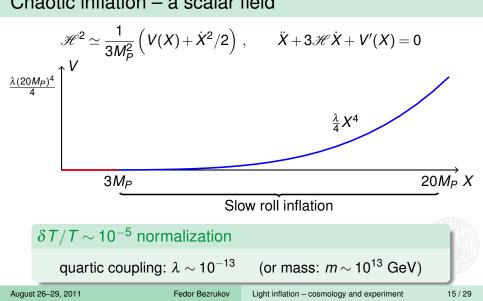
(Modification only in the gravity sector)

August 26-29, 2011

Inflationary model

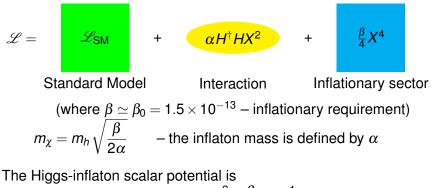
Bounds from cosmology - inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Chaotic inflation – a scalar field



Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Light inflaton model adds one scalar particle to the SM



$$V(H,X) = \lambda \left(H^{\dagger}H - \frac{\alpha}{\lambda}X^{2}\right)^{2} + \frac{\beta}{4}X^{4} - \frac{1}{2}\mu^{2}X^{2} + V_{0}$$

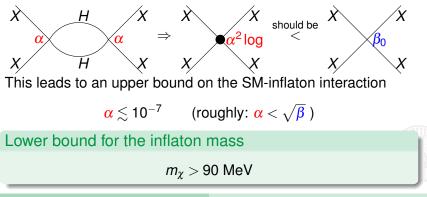
[Anisimov, Bartocci, FB'08, FB, Gorbunov'09]

August 26–29, 2011

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Radiative corrections require a small SM-inflaton coupling

Radiative corrections induce quartic coupling which should not spoil the flatness of the potential



Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Radiative corrections require a small SM-inflaton coupling

Radiative corrections induce quartic coupling which should not spoil the flatness of the potential

This leads to an upper bound on the SM-inflaton interaction

$$lpha \lesssim 10^{-7}$$
 (roughly: $lpha < \sqrt{eta}$)

Lower bound for the inflaton mass

$$m_\chi > 90 \; {
m MeV}$$

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Preheating requires large SM-inflaton coupling

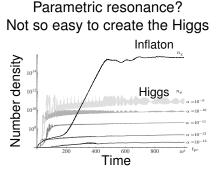
- After inflation: empty & cold
- Needed: hot, *T_r* > 150 GeV (to get baryogenesis)

Equating *H* production rate $(\propto \alpha^2)$ to Hubble expansion rate $(\propto T^2) \Gamma_{XX \to HH} \sim \mathcal{H}$

Lower bound on α

 $lpha\gtrsim7 imes10^{-10}$

[Anisimov, Bartocci, FB'08]



The large Higgs self interaction destroys coherence and spoils parametric resonance.

August 26-29, 2011

Fedor Bezrukov

Light inflation - cosmology and experiment

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Inflaton is in the experimentally explorable range

Inflaton mass window (from Cosmology)

90 MeV $< m_{\chi} <$ 1.8 GeV

Lower bound: radiative corrections

Upper bound: sufficient reheating

Also possible: $2m_H < m_\chi \lesssim 600$ GeV

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Inflaton-SM Interactions

As the Higgs boson, but light and suppressed by $\theta = \sqrt{2\beta} v/m_{\chi}$

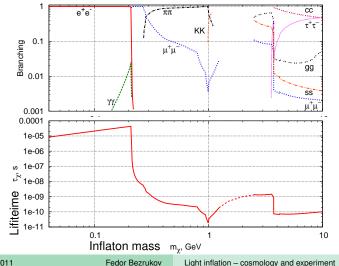
- Created: in meson decays
- Decays: the heaviest particle pairs (*ee*, $\pi\pi$, $\mu\mu$, *KK*)
- Interacts with media: extremely weakly

$$\begin{aligned} \mathscr{L}_{\chi\bar{f}f} &= \theta \, \frac{m_{f}}{v} \, \chi \bar{f}f = \sqrt{2\beta} \, \frac{m_{f}}{m_{\chi}} \chi \bar{f}f \\ \mathscr{L}_{\chi\pi\pi} &= 2\kappa \sqrt{2\beta} \cdot \frac{\chi}{m_{\chi}} \cdot \left(\frac{1}{2} \partial_{\mu} \pi^{0} \partial^{\mu} \pi^{0} + \partial_{\mu} \pi^{+} \partial^{\mu} \pi^{-}\right) \\ &- (3\kappa + 1) \sqrt{2\beta} \cdot \frac{\chi}{m_{\chi}} \cdot m_{\pi}^{2} \cdot \left(\frac{1}{2} \pi^{0} \pi^{0} + \pi^{+} \pi^{-}\right) \qquad \left(\kappa = 2/9\right) \\ \mathscr{L}_{\chi\gamma\gamma} &\approx \frac{F_{\gamma\gamma}\alpha}{4\pi} \, \frac{\sqrt{2\beta}}{m_{\chi}} \, \chi \, F_{\mu\nu} F^{\mu\nu} \qquad \qquad \mathscr{L}_{\chi gg} \approx \frac{F_{gg}\alpha_{s}}{4\sqrt{8}\pi} \, \frac{\sqrt{2\beta}}{m_{\chi}} \, \chi \, G_{\mu\nu}^{a} \, G^{a\mu\nu} \end{aligned}$$

August 26–29, 2011

Inflationary model Bounds from cosmology - inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Inflaton is relatively long lived



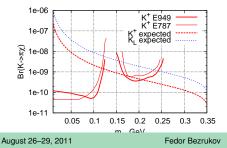


August 26-29, 2011

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Production: hadron decays

$$\begin{array}{c} \mathsf{Br}\left(\mathcal{K}^{+} \to \pi^{+}\chi\right) \approx \ 2.3 \times 10^{-9} \\ \\ \mathsf{Br}\left(\mathcal{K}_{L} \to \pi^{0}\chi\right) \approx \ 1.0 \times 10^{-8} \\ \\ \mathsf{Br}\left(\eta \to \pi^{0}\chi\right) \approx 1.8 \times 10^{-12} \\ \\ \mathsf{Br}\left(B \to X_{s}\chi\right) \approx \ 10^{-5} \end{array} \right\} \times \left(\frac{\beta}{\beta_{0}}\right) \cdot \left(\frac{100 \text{ MeV}}{m_{\chi}}\right)^{2} \cdot \left(\frac{m_{\chi}}{m_{hadron}}\right)$$

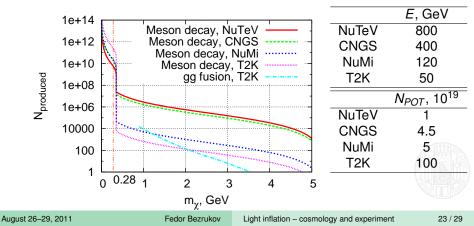


Bound from
 $\mathcal{K}^+ \to \pi^+ + \text{nothing}$ $m_\chi > 120 \text{ MeV}$ Disfavoured:
 $170 \text{ MeV} \lesssim m_\chi \lesssim 205 \text{ MeV}$

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

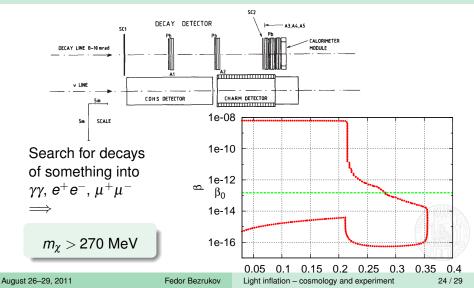
Production: beam dump, ideal luminosity

$$\frac{\sigma}{\sigma_{pp,\text{total}}} = M_{pp} \Big(\chi_s(0.5 \operatorname{Br}(K^+ \to \pi^+ \chi) + 0.25 \operatorname{Br}(K_L \to \pi^0 \chi)) + \chi_c \operatorname{Br}(B \to \chi X_s) \Big)$$



Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Beam dump: CHARM bound is the best at present



Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Production: search in B decays

$$\begin{array}{l} \operatorname{Br}(\mathcal{K}^{+} \to \pi^{+} \chi) \approx 2.3 \times 10^{-9} \\ \operatorname{Br}(\mathcal{K}_{L} \to \pi^{0} \chi) \approx 1.0 \times 10^{-8} \\ \operatorname{Br}(\eta \to \pi^{0} \chi) \approx 1.8 \times 10^{-12} \\ \operatorname{Br}(\mathcal{B} \to \mathcal{X}_{s} \chi) \approx 10^{-5} \end{array} \times \left(\frac{\beta}{\beta_{0}}\right) \cdot \left(\frac{100 \text{ MeV}}{m_{\chi}}\right)^{2} \cdot \left(\frac{m_{\chi}}{m_{hadron}}\right)$$

- Inflaton is produced quite abundant in *B* decays
- With typical lifetime of 10⁻⁹ s it decays at some distance but inside the detector
- Search for events with offset vertex in b-factories BaBar, Belle
- LHCb !

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Validity up to inflationary scale

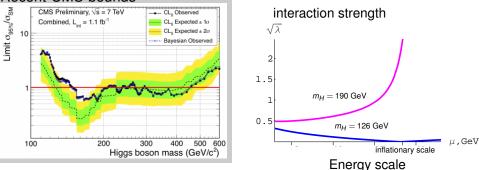
interaction strength $\sqrt{\lambda}$ 2 Radiative corrections – "screening" of the Higgs 1.5 m_H = 190 GeV self-interaction depending on 1 scale 0.5 m_H = 126 GeV μ,GeV inflationary scale Energy scale Higgs mass bounds

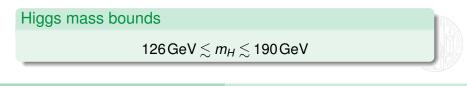
$126\,{ m GeV} \lesssim m_H \lesssim 190\,{ m GeV}$

August 26–29, 2011

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Validity up to inflationary scale Recent CMS bounds





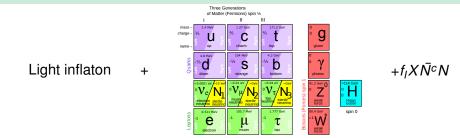
August 26-29, 2011

Fedor Bezrukov

Light inflation - cosmology and experiment

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Dark matter – add vMSM and stir



[Asaka, Blanchet, Shaposhnikov'05, Shaposhnikov, Tkachev'06]

- DM sterile neutrinos are produced in inflaton decays
- BAU via leptogenesis with two heavier sterile neutrinos

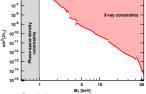
DM neutrino mass bound from production mechanism

 $M_1 \lesssim 80 {\rm keV}$

Inflationary model Bounds from cosmology – inflation and reheating Experimental detection of the inflaton Higgs mass bounds

Possible search for vMSM neutrino in the lab and in the Universe

- DM sterile neutrino N_1 , $M_1 \sim 1-80$ keV
 - X-ray line from the DM radiative decay $N_1 \rightarrow v \gamma$
 - Neutrinoless double beta decay $m_{ee} < 50 \times 10^{-3} \text{ eV}$



- Lepton asymmetry generating N_{2,3}, M_{2,3} ~ GeV
 - Neutrino production hadron decays: kinematics
 - Missing energy in K decays
 - Peaks in momentum of charged leptons for two body decays
 - Neutrino decays into SM particles: "nothing" to leptons and hadrons
 - Beam target experiments with high intensity proton beam, detector (preferably not dense) after the shielding.

[D. Gorbunov, M.Shaposhnikov'07]



Start from:

- Explain every experimental fact
- Expand the Standard Model in a minimal way

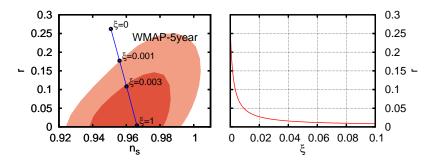
Arrive to:

- Predictions for low energy experiments!
- Model with additional scalar inflaton
 - Inflaton is light, $90 \,\mathrm{MeV} < m_{\chi} < 1.8 \,\mathrm{GeV}$
 - Higgs boson mass is in the window 126 190 GeV
 - Inflaton can be searched in rare decays! (LHCb)



Based on

WMAP-5 bounds



Message

With non-minimal coupling it is very natural for $\beta \phi^4$ inflation to be compatible with observations!

Dark matter – add vMSM and stir

A vMSM inspired model with inflation χ (Shaposhnikov&Tkachev'06)

$$\mathcal{L} = (\mathcal{L}_{SM} + \bar{N}_l i \partial_\mu \gamma^\mu N_l - F_{\alpha l} \bar{L}_\alpha N_l \Phi - \frac{\hbar}{2} \bar{N}_l^c N_l X + \text{h.c.}) + \frac{1}{2} (\partial_\mu X)^2 - V(\Phi, X)$$

$$\Omega_N = \frac{1.6f(m_\chi)}{S} \cdot \frac{\beta}{1.5 \times 10^{-13}} \cdot \left(\frac{M_1}{10 \text{keV}}\right)^3 \cdot \left(\frac{100 \text{ MeV}}{m_\chi}\right)^3 \,,$$

DM sterile neutrino mass bound
$$M_1 \lesssim 13 \cdot \left(\frac{m_{\chi}}{300 \text{ MeV}}\right) \left(\frac{S}{4}\right)^{1/3} \cdot \left(\frac{0.9}{f(m_{\chi})}\right)^{1/3} \text{keV} .$$

August 26-29, 2011

~

Parametric enchancement

Let us suppose again that there is an inflaton X coupled to some particle ϕ . Then, during inflaton oscillations, for the ϕ modes with momentum k we have

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2(t)} + g^2X(t)^2\right)\phi_k = 0$$

- Important -X(t) oscillates
- Let us neglect the Universe expansion, and say that $X(t) = A\sin(\omega t)$, then

Mathieu equation

$$\frac{d^2\phi_k}{d\eta^2} + (A_k - 2q\cos 2\eta) = 0$$

where
$$A_k = k^2/\omega^2 + 2q$$
, $q = g^2 X_0^2/4\omega^2$, $\eta = \omega t$.

August 26-29, 2011

Fedor Bezrukov

Light inflation - cosmology and experiment

Temperature estimate for the reheating

Equating mean free path $n\sigma_{2I \rightarrow 2H} v \sim n \frac{\alpha^2}{\pi p_{avg}^2}$ with the Hubble rate $H = \frac{T^2}{m_{Pl}} \sqrt{\frac{\pi^2 g_*}{90}}$ we get $T_R \approx \frac{\zeta(3)\alpha^2}{\pi^4} \sqrt{\frac{90}{g_*}} m_{Pl}$

Requiring $T_R > 150 \,\text{GeV}$ we can obtain the lower bound on α $\alpha \ge 7.3 \times 10^{-8}$,

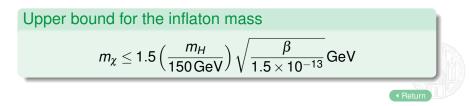


Temperature estimate for the reheating II

However,
$$p_{\text{avg}} \sim T$$
, the cross-section is enhanced, so

$$\frac{\zeta(3)\alpha^2}{\pi^3} \frac{T^4}{p_{\text{avg}}^3} \sim \frac{T^2}{\sqrt{\frac{90}{8\pi^3 g^*}}} M_{Pl}$$

For this estimate the bound is *weaker* $\alpha \ge 7 \times 10^{-10}$



Inflaton mass window

Flatness from radiative corrections

$$m_{\chi} > 120 \left(rac{m_h}{150 \ {
m GeV}}
ight) \left(rac{eta}{1.5 imes 10^{-13}}
ight)^{rac{1}{2}} \ {
m MeV}$$

Sufficient reheating

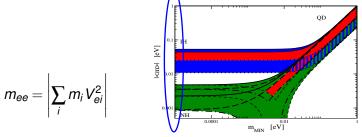
$$m_{\chi} \leq 1.5 \left(rac{m_H}{150\,\mathrm{GeV}}
ight) \left(rac{eta}{1.5 imes 10^{-13}}
ight)^{rac{1}{2}}\,\mathrm{GeV}$$

To be precise, the window also exists

$$2m_H < m_\chi \lesssim 460 \cdot \left(\frac{m_h}{150 \text{ GeV}}\right)^{4/3} \cdot \left(\frac{\beta}{1.5 \times 10^{-13}}\right)^{1/3} \text{ GeV}$$



$0\nu\beta\beta$ effective Majorana mass is small



- contribution from N_1 is negligible $|M_1 \theta_{e1}^2| \le 10^{-5} \text{ eV}$
- For heavier active neutrinos the contribution is always negative $m_{ee} < |\sum_i m_i V_{ei}^2|$ smaller prediction

$$m_{ee} < 50 imes 10^{-3} {
m eV}$$

F.B., 2006

$0\nu\beta\beta$ effective Majorana mass is small $m_{ee} = \left|\sum_{i} m_{i}V_{ei}^{2}\right|$

- contribution from N_1 is negligible $|M_1 \theta_{e1}^2| \le 10^{-5}$ eV
- For heavier active neutrinos the contribution is always negative $m_{ee} < |\sum_i m_i V_{ei}^2|$ smaller prediction

$$m_{ee} < 50 \times 10^{-3} \text{ eV}$$

F.B., 2006

Field dependent cut-off makes the model consistent



August 26-29, 2011

Inflationary regime - EW chiral theory

$$\mathcal{L}_{chiral} = \frac{1}{2} (\partial_{\mu} \chi)^2 - U(\chi)$$
$$- \frac{1}{2g^2} tr[W_{\mu\nu}^2] - \frac{v^2}{4} tr[V_{\mu}^2]$$
$$+ i \bar{Q}_{L,R} \not D Q_{L,R} - (\frac{y_t v}{\sqrt{2}} \bar{Q}_L \tilde{\mathscr{U}} Q_R + \dots + h.c.)$$

with

$$\mathscr{U} = \exp\left[2i\pi^{a}T^{a}\right], \quad V_{\mu} = (\partial_{\mu}\mathscr{U})\mathscr{U}^{\dagger} + iW_{\mu} - i\mathscr{U}B_{\mu}^{Y}\mathscr{U}^{\dagger}$$

and

$$v^{2} = rac{h^{2}}{\Omega^{2}(h)} = rac{M_{P}^{2}}{\xi} \left(1 - e^{-2\chi/\sqrt{6}M_{P}}
ight)^{-1}$$

RG equations in the inflationary regime

$$16\pi^{2}\mu\frac{\partial}{\partial\mu}g' = \left(\frac{1}{6} - \frac{1}{12} + \frac{20n_{f}}{9}\right)g'^{3}, \qquad (1)$$

$$16\pi^{2}\mu\frac{\partial}{\partial\mu}g = -\left(\frac{43}{6} + \frac{1}{12} - \frac{4n_{f}}{3}\right)g^{3}.$$
 (2)

$$16\pi^2 \mu \frac{\partial}{\partial \mu} g_3 = -7g_3^2 \,. \tag{3}$$

$$16\pi^2 \mu \frac{\partial}{\partial \mu} \xi = -\left(\frac{3}{2}g'^2 + 3g^2 - 6y_t^2\right)\xi \ . \qquad (v^2 \propto 1/\xi) \qquad (4)$$

$$16\pi^{2}\mu\frac{\partial}{\partial\mu}y_{t} = \left(-\frac{17}{12}g'^{2} - \frac{3}{2}g^{2} - 8g_{3}^{2} + 3y_{t}^{2}\right)y_{t}.$$

$$16\pi^{2}\mu\frac{\partial}{\partial\mu}\left(\frac{\lambda}{\xi^{2}}\right) = \frac{1}{\xi^{2}}\left(-6y_{t}^{4} + \frac{3}{8}\left(2g^{2} + (g'^{2} + g^{2})^{2}\right)\right).$$
 (6)

(5)

Backup slides

Effective potential

can be obtained from the SM one by

- removing the terms corresponding to the Higgs scalar loops
- setting Goldstone boson masses to zero



- FB, M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008)
- FB, D. Gorbunov, M. Shaposhnikov, JCAP **06**, 029 (2009)
- FB, A. Magnin, M. Shaposhnikov, Phys. Lett. B 675, 88 (2009)
- FB, A. Magnin, M. Shaposhnikov, S. Sibiryakov, JHEP 1101, 016 (2011).
- FB, M. Shaposhnikov, JHEP 0907 (2009) 089
- A. Anisimov, Y. Bartocci, F. L. Bezrukov, Phys. Lett. B 671, 211 (2009)
- FB, D. Gorbunov, JHEP **05** (2010) 010
- T. Asaka, M. Shaposhnikov, Phys. Lett. B 620 (2005) 17
- T. Asaka, S. Blanchet, M. Shaposhnikov, Phys. Lett. B 631 (2005) 151
- M. Shaposhnikov, I. Tkachev, Phys. Lett. B 639 (2006) 414
- A. A. Starobinsky, Phys. Lett. **B91** (1980) 99-102
 - D. S. Gorbunov, A. G. Panin, [arXiv:1009.2448 [hep-ph]]

