Multi-Scalar Models and Minimal Flavour Violation

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based on work done in collaboration with

F. J. Botella, M. N. Rebelo, M. N. Nebot

and earlier work with

W. Grimus and L. Lavoura

- Neutral Currents have played a crucial rôle in the construction of the SM and its experimental tests.
- The discovery of Neutral weak currents was the first great success of the SM
- An important feature of Flavour-Changing-Neutral Currents (FCNC) :

  They are forbidden at tree level, both in the SM and in most of its extensions
- EPS prize to Gargamelle collaboration in 2009
- EPS prize to GIM in 2011.

At loop level, FCNC are generated and have played a crucial rôle in testing the SM, and in putting bounds on New Physics beyond the SM.

- Neutral meson mixings: $K^0 - \overline{K^0}$, $D^0 - \overline{D^0}$, $B_d^0 - \overline{B_d^0}$, $B_s^0 - \overline{B_s^0}$
- rare kaon decays, rare b-meson decays,
- CP violation

SM contributes to these processes at loop level

$$\Downarrow$$

New Physics has a chance to give significant contributions.

The need to suppress FCNC led to two dogmas:

- No Z-mediated FCNC at tree level
- No FCNC in the scalar sector, at tree level

Glashow and Weinberg (PRD 1977)
E. A. Paschos (PRD, 1977)
derived necessary and sufficient conditions:

(i) All quarks of fixed charge and helicity must transform according to the same irreducible representation of SU(2) and correspond to the same eigenvalue of T_3

(ii) All quarks should receive their contributions to the quark mass matrix from a single neutral scalar VeV

- Can one violate these two dogmas in "reasonable" extensions of the SM? Yes! "reasonable" means that FCNC should be naturally suppressed without fine-tuning.

- In the gauge-sector, the dogma can be violated through the introduction of a $Q = -1/3$ and/or a $Q = 2/3$ vector-like quark

$\Downarrow$

Naturally small violations of 3×3 unitarity of V^{CKM}

$\Downarrow$

Z-mediated, Naturally suppressed FCNC at tree level.

Example : Addition of one $Q=-1/3$ vector-like quark, to the SM :

$D_L, D_R \rightarrow$ singlets under $SU(2)_L$ (very large number of references)

Charged currents :

$$(\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uD} \\ V_{cd} & V_{cs} & V_{cb} & V_{cD} \\ V_{td} & V_{ts} & V_{tb} & V_{tD} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}_L W_\mu + h.c.$$

Non-orthogonality of columns of V leads to terms like :

$$\frac{g}{\cos\theta_W} Z_{bd}\, \bar{b}_L \gamma_\mu d_L Z^\mu$$

$$Z_{bd} = V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^*$$

$Z_{bd} \rightarrow$ suppressed by $(m/M)^2$

m – mass of standard quarks

M – mass of D quark

Some comments on models with vector-like quarks:

- Lead to naturally suppressed, but non-vanishing tree level FCNC, mediated by Z_μ

- Lead to naturally suppressed deviations of 3×3 unitarity of V_{CKM}.

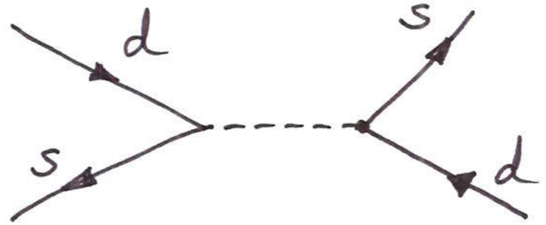
3×3 unitarity of V^{CKM} has to be checked experimentally!!

- Nothing "strange" in having deviations of 3×3 unitarity of V_{CKM}. The PMNS matrix in the leptonic sector, in the context of type-one seesaw, is not 3×3 unitary.
- Vector-like quarks provide the simplest model with spontaneous CP violation, with a complex V_{CKM}, in agreement with experiment.
- Provide a framework to have a Common Origin for all CP Violations! (See M.N. Rebelo's talk)

Scalar Sector

Can one have scalar-mediated FCNC at tree level, but somehow suppressed by "small V^{CKM} elements"?

Most dangerous couplings:



$K_L - K_S$ mass difference $\Rightarrow$ $m_H \gtrsim 1\,\text{TeV}$

CP violation (ε_K) $\Rightarrow$ $m_H \gtrapprox 30\,\text{TeV}$

The possibility that FCNC could be suppressed by small V^{CKM} elements was considered by various authors:

- L. Hall, S. Weinberg
- A. Antaramian, L. Hall, A. Rasin
- Yoshipura, S. D. Rindani

Interesting, but ad-hoc assumptions, not based on an exact (or softly broken) symmetry of the Lagrangian.

Question : Can one have a multi-scalar extension of the SM where, as a result of a symmetry of the Lagrangian, there are FCNC at tree level, but all couplings controlled by V^{CKM}, without any other flavour parameters? Answer : Yes!!

- First we show that this looks like "an impossible mission"
- Then we show that there are models which fulfill the above condition, but the number of models is severely restricted

G. C. B, W. Grimus, L. Lavoura, Phys. Lett. 1996
F. Botella, G.C.B., MN Rebelo, Phys. Lett. B (2010)
F. Botella, G.C.B., M.N. Rebelo, M. Nebot, arxiv: 1102.0520 [hep-ph]

- What is the motivation for considering multi-scalar models with FCNC? It is likely that a theory of flavour will involve various scalar doublets.
- It is desirable that a correct theory of flavour predicts new phenomena, making some testable predictions. A possible prediction could be the existence of non-vanishing but naturally suppressed scalar-mediated FCNC
- Nature may, once more, surprise us:

  Recent "surprises" with flavour:

  — A heavy top

  — Large leptonic mixing

The requirement of rephasing invariance

Let us consider a FCNC transition connecting a quark d_j to another quark d_k. The transition could be mediated by a scalar or by a vector boson:

$$\mathcal{L}_{scalar} = \bar{d}_{Lj}\, \Gamma^{S}_{jk}\, d_{Rk}\, S \;;\; \mathcal{L}_{vector} = \bar{d}_{Lj}\, \Gamma^{V}_{jk}\, \gamma_\mu\, d_{Lk}\, V^\mu$$

Γ^S, Γ^V may arise at tree level or in higher orders.
Assume that d_j denotes quark mass eigenstates.
Under rephasing of quark fields:

$$d_j \rightarrow d'_j = \exp(-i\beta_j)\, d_j$$

Γ^S, Γ^V have to transform in such a way that the above interactions remain invariant. This implies that under rephasing

$$\Gamma_{jk} \rightarrow \Gamma'_{jk} = \exp\left[i(\beta_k - \beta_j)\right] \Gamma_{jk}$$

If we require that the flavour dependence of Γ_{jk} be completely controlled by V^{CKM}, this severely restricts the functional dependence of Γ_{jk} on V_{CKM}. The simplest forms allowed by rephasing invariance are :

$$\Gamma_{jk} = \sum_{\alpha} c_\alpha V_{\alpha j} V^*_{\alpha k}$$

where c_α are coefficients which are invariant under rephasing.

Note that the form of Z_{bd}, found previously, satisfies this requirement (as it had to !!) with

$$c_\alpha = 1 \quad \text{for all } c_\alpha$$

$$Z_{bd} = V_{ud} V^*_{ub} + V_{cd} V^*_{cb} + V_{td} V^*_{tb}$$

The case of Two scalar doublets models

Yukawa Interactions

$$\mathcal{L}_Y = - \overline{Q}^\circ_L \Gamma_1 \phi_1 d^\circ_R - \overline{Q}^\circ_L \Gamma_2 \phi_2 d^\circ_R - \overline{Q}^\circ_L \Delta_1 \tilde{\phi}_1 u^\circ_R - \overline{Q}^\circ_L \Delta_2 \tilde{\phi}_2 u^\circ_R + h.c.$$

$Q^\circ_L \rightarrow$ left-handed doublets ; d°_R, u°_R right-handed singlet

So this is a two-scalar doublet model of type III

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2 \right) \; ; \quad M_u = \frac{1}{\sqrt{2}} \left(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2 \right)$$

Diagonalized by:

$$U^\dagger_{dL} M_d U_{dR} = D_d \equiv \text{diag.} (m_d, m_s, m_b)$$

$$U^\dagger_{uL} M_u U_{uR} = D_u \equiv \text{diag.} (m_u, m_c, m_t)$$

Expand Φ_j : $\Phi_j = e^{i\alpha_j}\begin{bmatrix} \phi_j^+ \\ 1/\sqrt{2}\,(v_j+\rho_j+i\eta_j)\end{bmatrix}$ $\quad j=1,2$

It is convenient to define new fields $G^+, G^0, H^\pm, H^0, R$:

$$\begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = O\begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix} \; ; \; \begin{bmatrix} G^0 \\ I \end{bmatrix} = O\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \; ; \; \begin{bmatrix} H^0 \\ R \end{bmatrix} = O\begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

where: $O = \frac{1}{v}\begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix}$; $v = \sqrt{v_1^2+v_2^2} = \left(\sqrt{2}\,G_F\right)^{-1/2} \approx 246\,\mathrm{GeV}$

$G^+, G^0 \rightarrow$ Goldstone bosons

$H^0, R, I \rightarrow$ neutral scalars

$H^+ \rightarrow$ charged scalar

It is convenient to write the Yukawa coupling in terms of quark mass eigenstates and the new fields $H^\pm, H^0, R, I$

$$\mathcal{L}_Y = \frac{\sqrt{2}\,H^+}{v}\,\bar{u}\left[V N_d \gamma_R + N_u^\dagger V \gamma_L\right] d_L + h.c. - \frac{H^0}{v}\left[\bar{u} D_u u + \bar{d} D_d d\right] -$$

$$- \frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^\dagger \gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^\dagger \gamma_L\right) d\right] +$$

$$+ i\,\frac{I}{v}\left[\bar{u}\left[N_u \gamma_R - N_u^\dagger \gamma_L\right] u - \bar{d}\left(N_d \gamma_R - N_d^\dagger \gamma_L\right) d\right]$$

$u, d \rightarrow$ quark mass eigenstates ; $\gamma_L = \frac{1}{2}(1-\gamma_5)$; $\gamma_R = \frac{1}{2}(1+\gamma_5)$

The matrices N_d, N_u are given by :

$$N_d = \frac{1}{\sqrt{2}}\, U_{dL}^\dagger \left[v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right] U_{dR} \; ; \; N_u = \frac{1}{\sqrt{2}}\, U_{uL}^\dagger \left[v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2\right] U_{uR}$$

Flavour-Changing Neutral couplings are controlled by the matrices N_d, N_u. For generic 2 scalar doublet models N_d, N_u are <u>arbitrary</u>!

It is convenient to write N_d in the following way:

$$N_d = \underbrace{\frac{v_2}{v_1} D_d}_{\text{Conserves flavour}} - \underbrace{\frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{dL}^{\dagger} e^{i\alpha} \Gamma_2 U_{dR}}_{\text{leads to FCNC}}$$

From the above expression for N_d, one concludes that there are Two Major Obstacles which one has to surmount inorder for N_d to be entirely controlled by V^{CKM}, with no free parameters:

(i) It is U_{dL} rather than the combination $U_{uL}^{\dagger} U_{dL} \equiv V^{CKM}$ which appears in N_d

(ii) How to get rid of the dependence on U_{dR}?

The first difficulty can be solved by means of a flavour symmetry constraining U_{uL} to have mixing only among two generations, for example:

$$U_{uL} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix}$$; In this case:

$$V^{CKM} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d_{31}} & U_{d_{32}} & U_{d_{33}} \end{bmatrix} = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d_{31}} & U_{d_{32}} & U_{d_{33}} \end{bmatrix}$$

So one has:

$$\boxed{\left(V^{CKM}\right)_{3j} = \left(U_{dL}\right)_{3j}}$$

In order to surmount difficulty (i) one has to further require that the flavour dependence of N_d on U_{dL} is only on the 3^{rd} row of U_{dL}.

How to surmount obstacle (ii), i.e. how to avoid the dependence on U_{dR}? Let us assume that Γ_2 is such that:

$$\Gamma_2 \propto P M_d \qquad \text{where } P \text{ is a fixed matrix.}$$

In this case:

$$U_{dL}^{+} \Gamma_2 U_{dR} \propto U_{dL}^{+} P M_d U_{dR} = U_{dL}^{+} P \underbrace{U_{dL} U_{dL}^{+}}_{\mathbb{1}} \overbrace{M_d U_{dR}}^{D_d} = U_{dL}^{+} P U_{dL} D_d$$

The flavour structure of Γ_1, Γ_2 should be such that a fixed matrix P exists satisfying:

$$\Gamma_2 \propto P M_d$$

One way of achieving this is by having:

$$P \Gamma_2 = k \Gamma_2 \quad ; \quad P \Gamma_1 = 0$$

Recall that $\quad M_d = 1/\sqrt{2} \left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2 \right)$

It has been shown (B., Grimus and Lavoura ≡ BGL) that it is possible to find a flavour symmetry of the Lagrangian such that it leads to a structure for Γ_i, Δ_i which imply FCNC at tree level, with strength completely controlled by V_{CKM}. BGL have imposed the following symmetry on the Lagrangian :

a) $Q^0_{Lj} \rightarrow \exp(i\alpha) Q^0_{Lj}$; $u^0_{Rj} \rightarrow \exp(i 2\alpha) u^0_{Rj}$ $\phi_2 \rightarrow \exp(i\alpha)\phi_2$

where $\alpha \neq 0, \pi$, with all other quark fields transforming trivially under the symmetry.

- The index j can be $j = 1, 2, 3$

Alternatively one can choose the symmetry:

b) $Q^0_{Lj} \rightarrow \exp(i\alpha) Q^0_{Lj}$ $d^0_{Rj} \rightarrow \exp(i 2\alpha) d^0_{Rj}$ $\phi_2 \rightarrow (-i\alpha)\phi_2$

Altogether 6 BGL models in the quark sector

Let us choose $j = 3$:

$$Q^{\circ}_{L3} \to \exp(i\alpha)\, Q^{\circ}_{L3} \; ; \; u^{\circ}_{R3} \to \exp(i2\alpha)\, u^{\circ}_{3} \; ; \; \phi_2 \to \exp(i\alpha)\, \phi_2$$

In this case the Yukawa matrices Γ_i, Γ_2 have the structure

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \; ; \; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} ; \; \Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

and $\quad PM_d = \frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 \quad ;$ with $P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

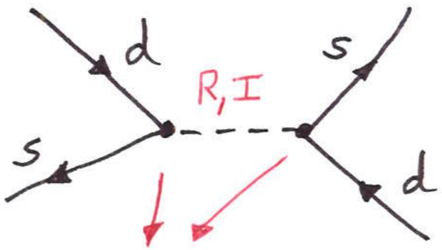
$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \left(V^{\dagger}_{CKM}\right)_{i3} \left(V_{CKM}\right)_{3j} (D_d)_{jj}$$

$$(N_u) = -\frac{v_1}{v_2}\, \mathrm{diag.}\,(0, 0, m_t) + \frac{v_2}{v_1}\, \mathrm{diag.}\,(m_u, m_c, 0)$$

In this example, the Higgs mediated FCNC are suppressed by the 3rd row of V^{CKM}. Furthermore, there are FCNC only in the down sector!

An important feature of the Model which we have described:

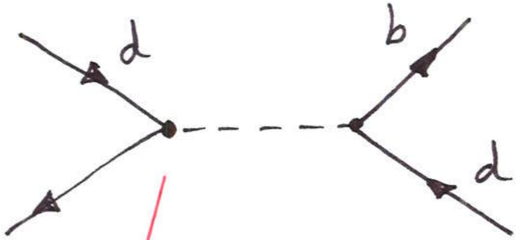
Strong and Natural Suppression of the "most dangerous processes"



$\Delta S = 2$ processes are strongly suppressed

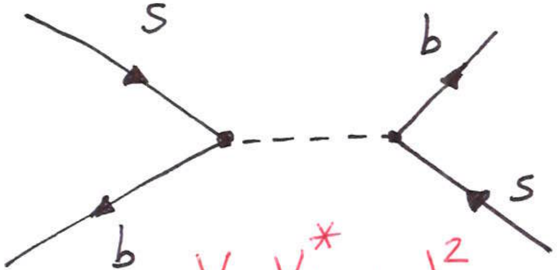
"light" Higgs are allowed

$V_{td} V_{ts}^* \sim \lambda^5 \Rightarrow$ altogether λ^{10} suppression!!



$\rightarrow$ may contribute significantly to $B_d - \overline{B}_d$ mixing

$V_{td} V_{tb}^* \sim \lambda^3$



$\rightarrow$ contribution to $B_s - \overline{B}_s$ mixing

$V_{ts} V_{tb}^* \sim \lambda^2$

An important Question:

Are there any other models, based on different abelian symmetries, leading also to FCNC at tree-level but completely controlled by V_{CKM}, without any further parameters? "Intuitive" answer: yes. Correct answer: No!!

Pedro Ferreira and João Silva (Phys. Rev. D 83 (2011)) have classified all possible implementations of an Abelian symmetry in two-scalar doublet models, imposing the request of having non-vanishing quark masses and not block-diagonal V^{CKM}. Answer: BGL are unique!!

This is a truly amazing result. We (BGL) did not use any systematic study, just used "intuition" and we were also Lucky!

So far, we have considered models which, due to the presence of a family symmetry, lead to FCNC, completely controlled by V^{CKM}. Question:

Can one make a "minimal-flavour-type" expansion of N_d, N_u?

It is clear that a necessary condition for N_d^0, N_u^0 to be of the "MFV type" is that they should be functions of M_d, M_u and no other flavour dependent couplings

The terms entering in the expansion of N_d^0, N_u^0 should have the right transformation properties under weak-basis (WB) transformations

Under a WB transformation, defined by :

$$Q^{\circ}_L \rightarrow W_L\, Q^{\circ}_L \quad ; \quad d^{\circ}_R \rightarrow W^d_R\, d^{\circ}_R \quad ; \quad u^{\circ}_R \rightarrow W^u_R\, u^{\circ}_R$$

The quark mass matrices M_u, M_d transform as:

$$M_d \rightarrow W_L^{\dagger} M_d W_R^d \quad ; \quad M_u \rightarrow W_L^{\dagger} M_u W_R^u$$

The matrices U_{dL}, U_{dR}, U_{uL}, U_{uR} transform as

$$U_{dL} \rightarrow W_L^{\dagger}\, U_{dL} \quad ; \quad U_{uL} \rightarrow W_L^{\dagger}\, U_{uL}$$

$$U_{dR} \rightarrow W_R^{d\dagger}\, U_{dR} \quad ; \quad U_{uR} \rightarrow W_R^{u\dagger}\, U_{uR}$$

while the Hermitian matrices $H_{d,u} \equiv M_{d,u} M_{d,u}^{\dagger}$ transform as:

$$H_d \rightarrow W_L^{\dagger} H_d W_L \quad ; \quad H_u \rightarrow W_L^{\dagger} H_u W_L$$

It is convenient to write H_d, H_u in terms of projection operators

(F. Botella, M. Nebot, O. Vives)

$$H_d = \sum_i m_{d_i}^2 P_i^{dL} \text{, where } P_i^{dL} = U_{dL} P_i U_{dL}^\dagger \text{, with } (P_i)_{jk} = \delta_{ij}\delta_{ik}$$

Obviously, under a WB transformation, N_d°, N_u° should transform as M_d, M_u. A MFV expansion for N_d°, N_u° with proper transformation properties is:

$$N_d^\circ = \lambda_1 M_d + \lambda_{2i} U_{dL} P_i U_{dL}^\dagger M_d + \lambda_{3i} U_{uL} P_i U_{uL}^\dagger \cdot M_d + \cdots$$

$$N_u^\circ = \tau_1 M_u + \tau_{2i} U_{uL} P_i U_{uL}^\dagger M_u + \tau_{3i} U_{dL} P_i U_{dL}^\dagger M_u + \cdots$$

In the mass eigenstate basis:

$$N_d^\circ = \lambda_1 D_d + \lambda_{2i} P_i D_d + \lambda_{3i} V_{CKM}^\dagger P_i V_{CKM} D_d + \cdots$$

with analogous expression for N_u°.

The BGL example considered before corresponds to the following truncation of our MFV expansion:

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_u^\dagger M_d$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_u$$

Note that the "truncation" corresponds to an exact symmetry of the Lagrangian.

Important point: Of the six BGL-type models only one is compatible with the MFV principle.

see: A. Buras, M. Carlucci, S. Gori, G. Isidori (1005.5310V1)

Extension to the Leptonic Sector

BGL models can be extended to the Leptonic Sector.

Consider implementation in the seesaw framework:

$$\mathcal{L}_{Y+\text{mass}} = - \bar{L}^0_L \Pi_1 \Phi_1 \ell^0_R - \bar{L}^0_L \Pi_2 \Phi_2 \ell^0_R - \bar{L}^0_L \Sigma_1 \tilde{\Phi}_1 \nu^0_R - \bar{L}^0_L \Sigma_2 \tilde{\Phi}_2 \nu^0_R + \frac{1}{2} \nu^{0T}_R C^{-1} M_R \nu^0_R + \text{h.c.}$$

$M_R \rightarrow$ right-handed Majorana mass matrix.

Impose the following Z_4 symmetry on the Lagrangian:

$$L^0_{L3} \rightarrow \exp(i\alpha) L^0_{L3} \;;\; \nu^0_{R3} \rightarrow \exp(i2\alpha) \nu^0_R \;;\; \Phi_2 \rightarrow e^{i\alpha} \Phi_2$$

$\alpha = \pi/2 \rightarrow$ choice dictated by the request of having a non-vanishing $\det M_R$

Structure of Leptonic mass matrices :

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Sigma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} \; ; \; M_R = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{bmatrix}$$

The non-vanishing of this entrie requires Z_4 !

Conclusions

- We live in the LHC ERA. At this stage "Nobody knows" what is the detailed mechanism of electroweak symmetry breaking, chosen by Nature.
- Multi-Scalar models may play an important rôle in solving the flavour puzzle. For that, it may be necessary to violate the Dogma of N.F.C. in the Scalar Sector. A theory of flavour may have its own mechanism for the suppression of FCNC in the Scalar Sector
- LHC may bring some surprises in the Scalar Sector for example "Non-Standard Higgs", hopefully giving us new hints to find a solution to the Flavour Puzzle