Is the standard Higgs boson a true massive field ?

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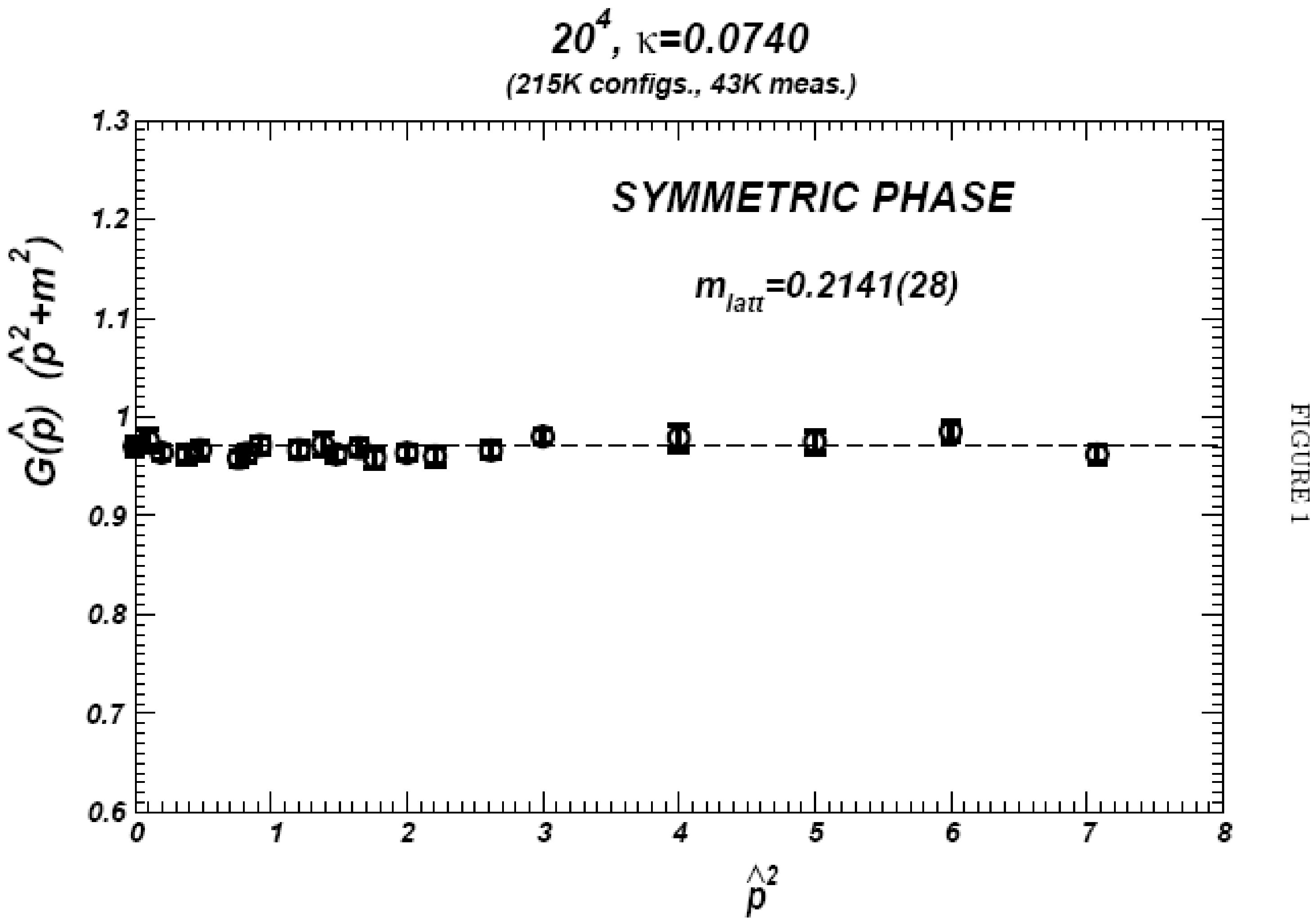
Abstract

1) Lattice calculations of the (connected) scalar propagator G (p)
2) Qualitative discrepancy between broken and symmetric phase for p→0
3) What does the theory say about G (p=0) ?
4) Perturbative calculations predict a standard massive Higgs boson
5) However: G (p=0) from the generating functional W[J]
   G (p=0) from rigorous RG approach
   G (p) from Stevenson's alternative calculation
   suggest that the standard Higgs boson is NOT a true massive field
6) Possible phenomenological implications: ultra-weak long-range forces

Lattice measurement of the scalar propagator

P. Cea et al. Mod.Phys.Lett.A 1999



Broken Phase (P. M. Stevenson Nucl. Phys. B 2005)

- Data for the connected propagator by Cea et al. Mod. Phys. Lett. A 1999

$$\zeta \equiv (\hat{p}^2 + m^2)G(p)$$

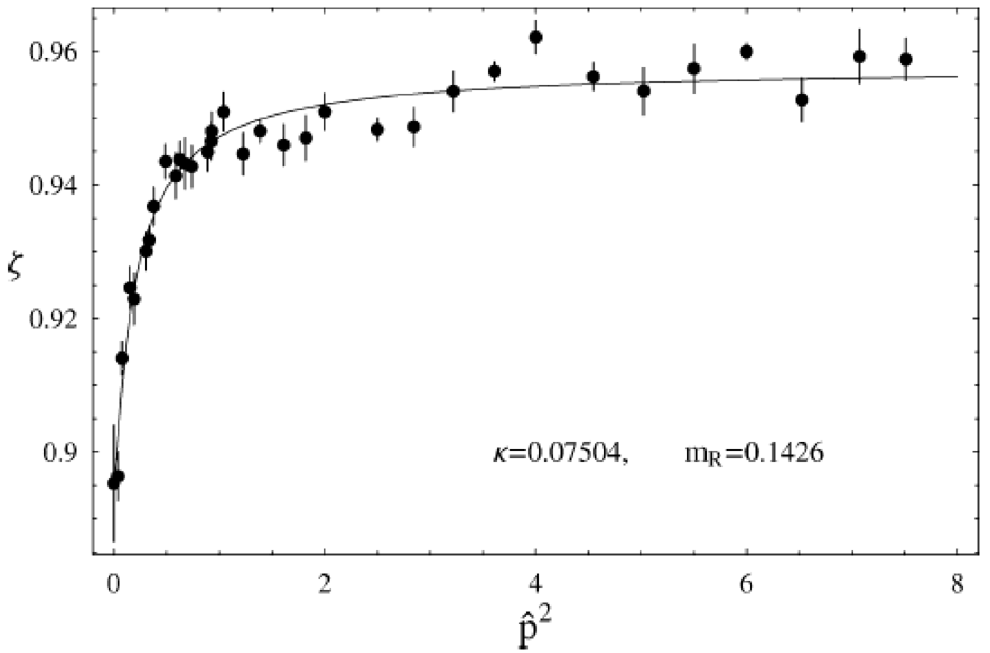
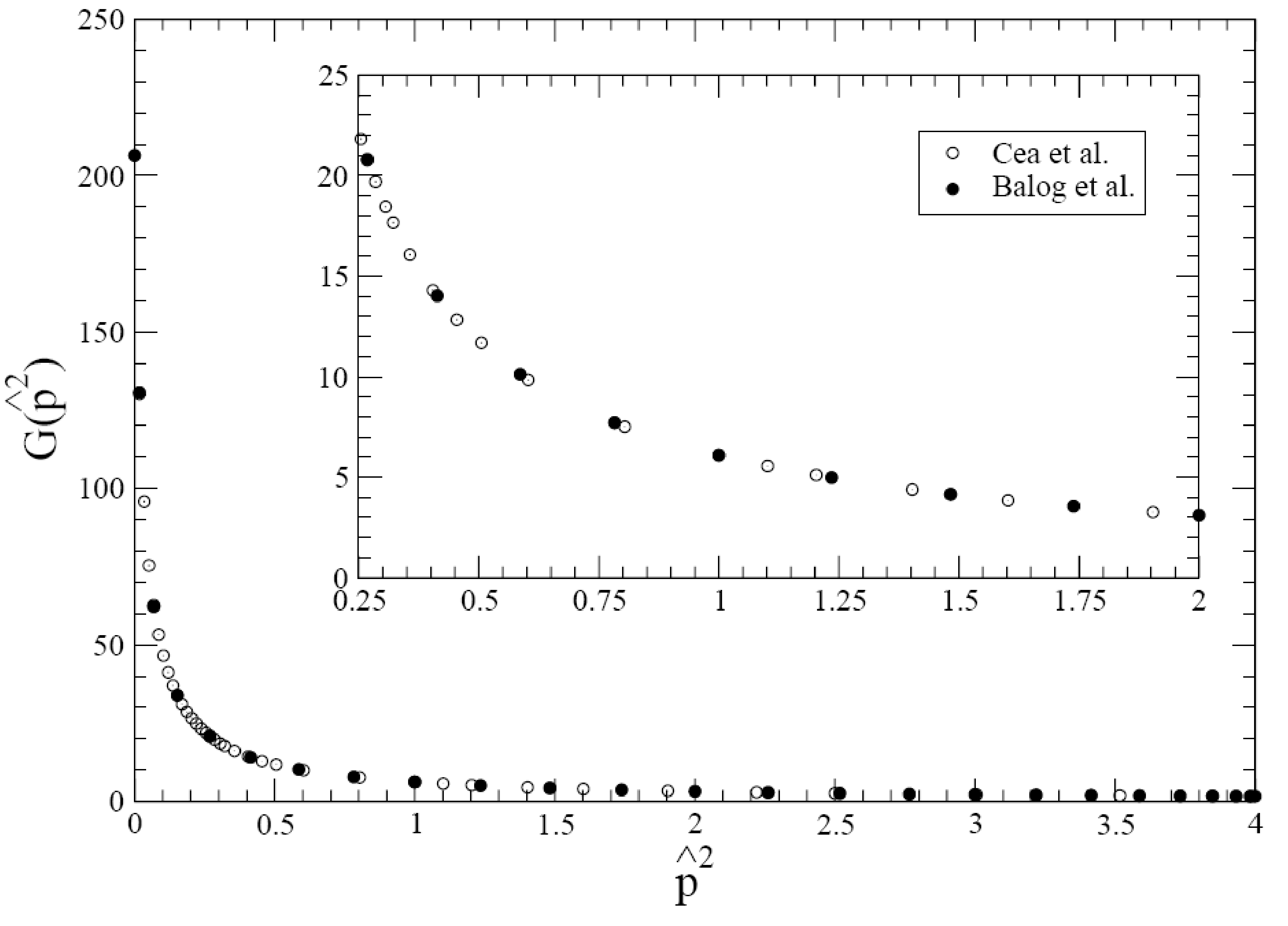


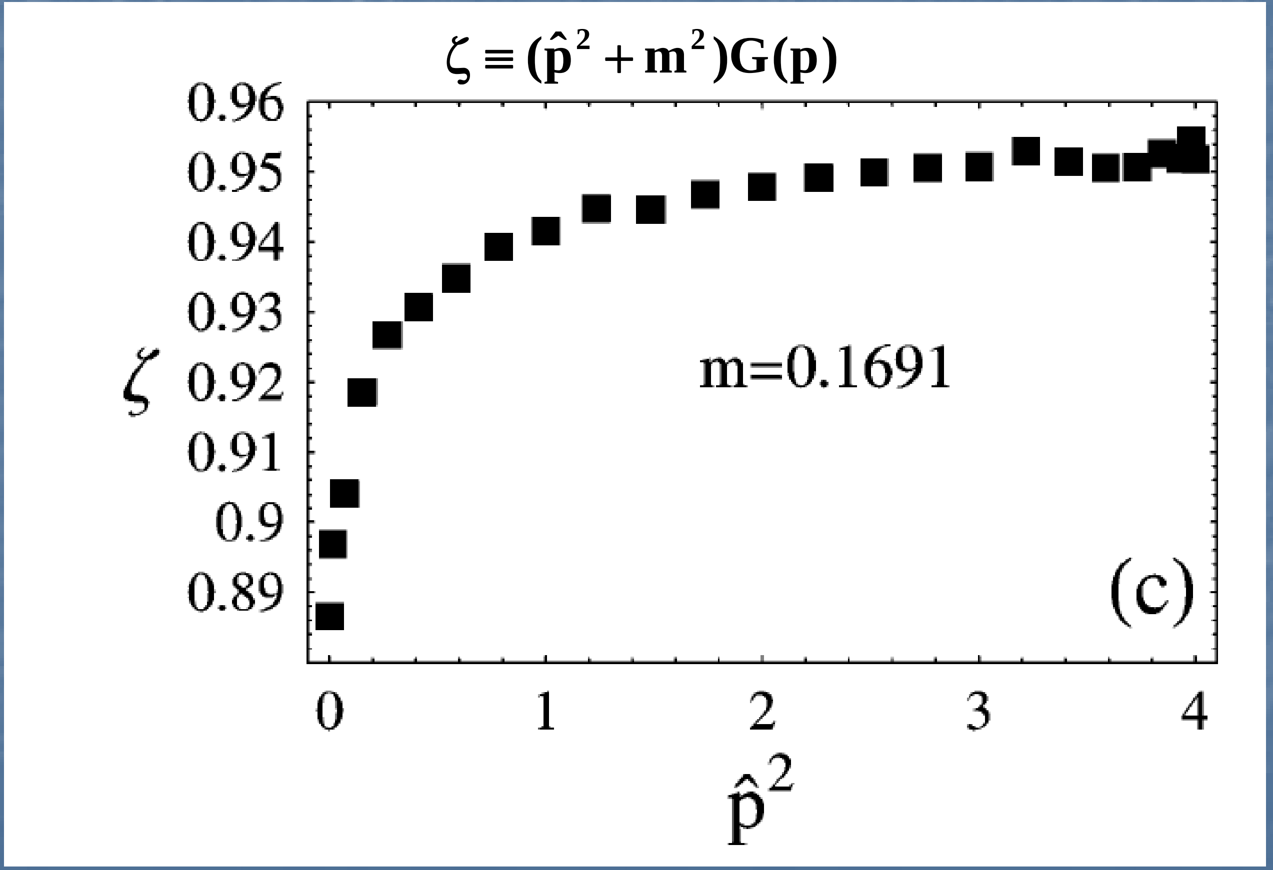
Fig. 9. As Fig. 7 but for $\kappa = 0.07504$, with $m_R = 0.1426$. The curve represents a simple, empirical fit to the data, to be used in Fig. 10.

Comparison with other authors



Broken Phase (P. M. Stevenson Nucl. Phys. B 2005)

- Propagator data by Balog et al. **Nucl. Phys. B 2005**



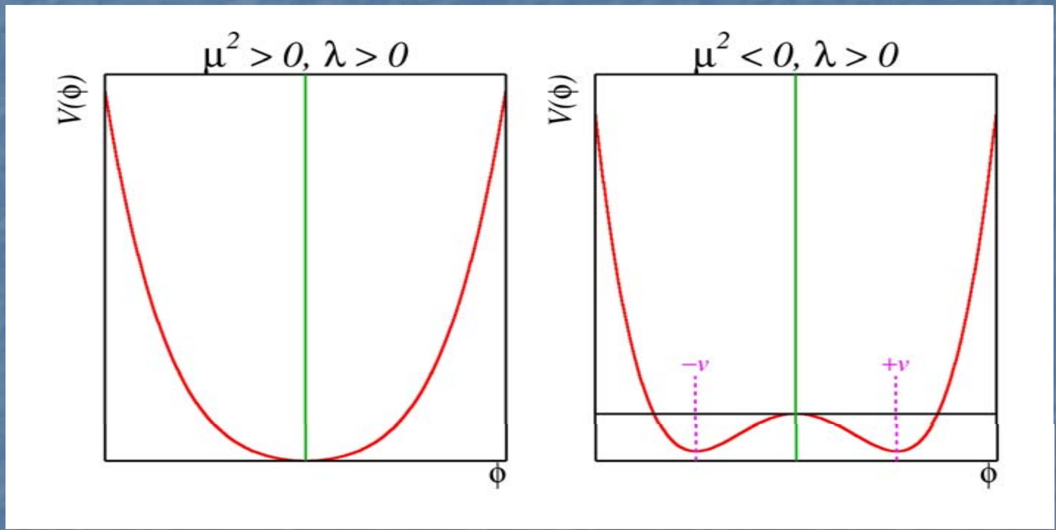
Summarizing

- Lattice simulations are not expected to fully display all properties of the infinite-volume theory
- Still, the existing calculations of the (connected) propagator G(p) provide qualitative differences between broken phase and symmetric phase for p→0
- Question: what does the theory say about G (p=0) ?

G (p=0) and the effective potential

- The 2-point function at p=0, the inverse of the connected propagator G (p=0), is obtained from the second derivative of the effective potential. Standard perturbative calculations give the following picture:

(a)



(b)

(a)

(b)

$$\mathbf{G^{-1}(p=0) = V''(\phi = 0)}$$

$$\mathbf{G^{-1}(p=0) = V''(\phi = \pm v)}$$

No qualitative difference between broken and symmetric phase. However…

G(p=0) from W[J]

- Most general description of SSB (M.C. : Phys. Rev. D 2002 ; Class. Quantum Grav. 2009). Consider an arbitrary classical action

$$S[\Phi;\Psi_1 ... \Psi_n]$$

depending on a scalar field $\Phi(x)$ and other n fields $\{\Psi_1(x), ... \Psi_n(x)\}$ (additional scalar fields, gauge fields, fermions,…)

The generating functional for the Green's functions of the Φ field is

$$Z[J] = \int [d\Phi(x)][d\Psi_1(x) ... d\Psi_n(x)] e^{\int d^4x J(x)\Phi(x) - S[\Phi;\Psi_1 ... \Psi_1]}$$

or by integrating formally on the $\{\Psi_1(x), ... \Psi_n(x)\}$

$$Z[J] = \int [d\Phi(x)] \, e^{\int d^4x J(x)\Phi(x) - S_{eff}[\Phi]}$$

SSB and the general structure of $S_{eff}[\Phi]$

- Take the average field in a large 4-volume Ω

and replace
$$\phi = \frac{1}{\Omega}\int d^4x\,\Phi(x)$$

so that
$$\Phi(x) = \phi + h(x)$$

By restricting to a constant source J(x)=J=const., one finds

$$Z(J) = \int_{-\infty}^{+\infty} d\phi\; e^{\Omega J\phi}\int [dh(x)]e^{-S_{eff}[\phi+h]}$$

Now, to any finite order in the loop expansion, the standard condition for SSB is expressed in terms of some non convex (NC='Non Convex') potential (see e. g. L. Maiani et al. Nucl. Phys. B 1986, U. Ritschel Phys. Lett. B 1993)

$$\int [dh(x)]e^{-S_{eff}[\phi+h]} = e^{-\Omega V_{NC}(\phi)}$$

$$Z(J) = \int_{-\infty}^{+\infty} d\phi \; e^{\Omega\left(J\phi - V_{NC}(\phi)\right)}$$

- The connected Green' functions at p=0 are obtained from

$$W(J) = \ln \frac{Z(J)}{Z(0)} \equiv \Omega w(J)$$

and one finds $\varphi = \dfrac{dw}{dJ}$ $\qquad G(p=0) = \dfrac{d^2 w}{dJ^2}$

In the saddle-point approximation, the results depend only on

1) the absolute minima of the non-convex potential $V_{NC}^{'}(\pm v) = 0$

2) its quadratic shape at the minima $V_{NC}^{''}(\pm v) \equiv m_h^2$

The results are

$$\varphi = v\left(\frac{J}{v m_h^2} + \tanh(\Omega J v)\right) \qquad G(p=0) = \frac{1}{m_h^2}\left(1 + \Omega m_h^2 v^2 [1 - \tanh^2(\Omega J v)]\right)$$

As it is well known, when J→0 , a non-zero v.e.v. requires to take the infinite-volume limit.

- By introducing dimensionless units

$$\varepsilon \equiv \frac{J}{v m_h^2} \qquad Y \equiv \Omega m_h^2 v^2$$

one gets

$$\varphi = v(\varepsilon + \tanh \varepsilon Y) \qquad G^{-1}(p=0) = \frac{m_h^2}{1 + Y(1 - \tanh^2 \varepsilon Y)}$$

Then, in the double limit where $\varepsilon \to \pm 0$ and $\varepsilon Y \to \pm\infty$ (so that $\varphi \to \pm v$) the zero-momentum connected propagator becomes a two-valued function. Example for $\varphi \approx +v$ by replacing

$$\tanh \varepsilon Y = \frac{\varphi}{v} - \varepsilon = 1 + \frac{\Delta\varphi}{v} - \varepsilon$$

1) $G^{-1}(p=0) \to m_h^2$ if $\Delta\varphi = |\Delta\varphi|$

2) $G^{-1}(p=0) \to 0$ if $\Delta\varphi = -|\Delta\varphi|$

Analogous results hold for $\varphi \approx -v$ depending on $\Delta\varphi = \pm|\Delta\varphi|$

- These two solutions admit a simple geometric interpretation as right- and left- second derivatives of the Legendre transformed effective potential

$$\mathbf{V_{LT}(\varphi) = [J\varphi - w(J)]_{J=J(\varphi)}}$$

- Due to its convexity, this is not an infinitely-differentiable function in the presence of SSB (K. Symanzik 1970).
- Thus the issue about G(p=0) requires to understand which is the most appropriate definition of the effective potential,

  $\mathbf{V_{NC}(\varphi)}$ or $\mathbf{V_{LT}(\varphi)}$?

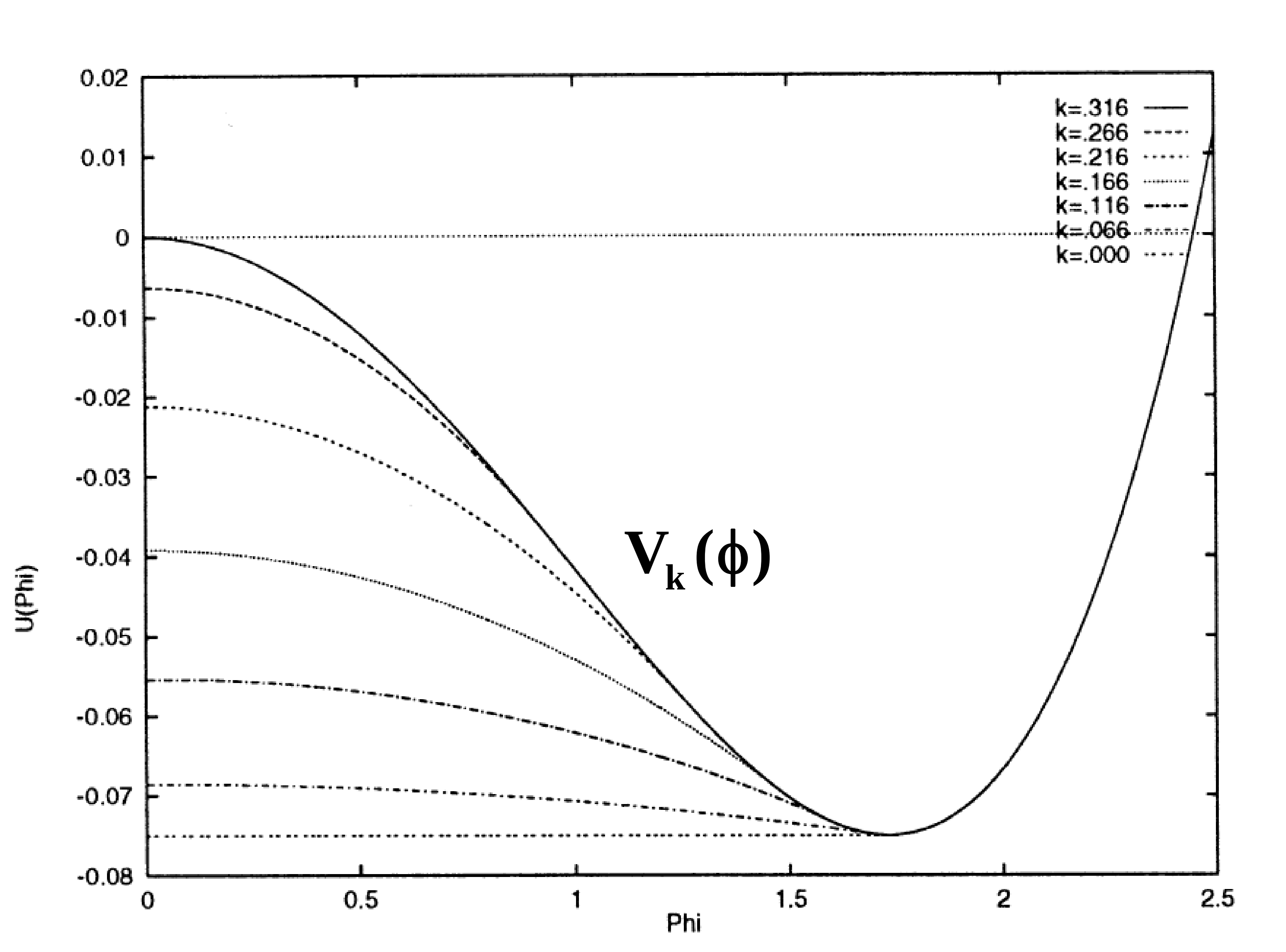
The effective potential from rigorous RG approach

- A widely accepted approach to Renormalization Group consists in starting from a bare action defined at some ultraviolet cutoff Λ and effectively integrating out shells of quantum modes down to an infrared cutoff $\mathbf{k}$. This procedure provides a k-dependent effective action $\Gamma_k[\Phi]$ that evolves into the full effective action in the $\mathbf{k} \to \mathbf{0}$ limit, i.e.

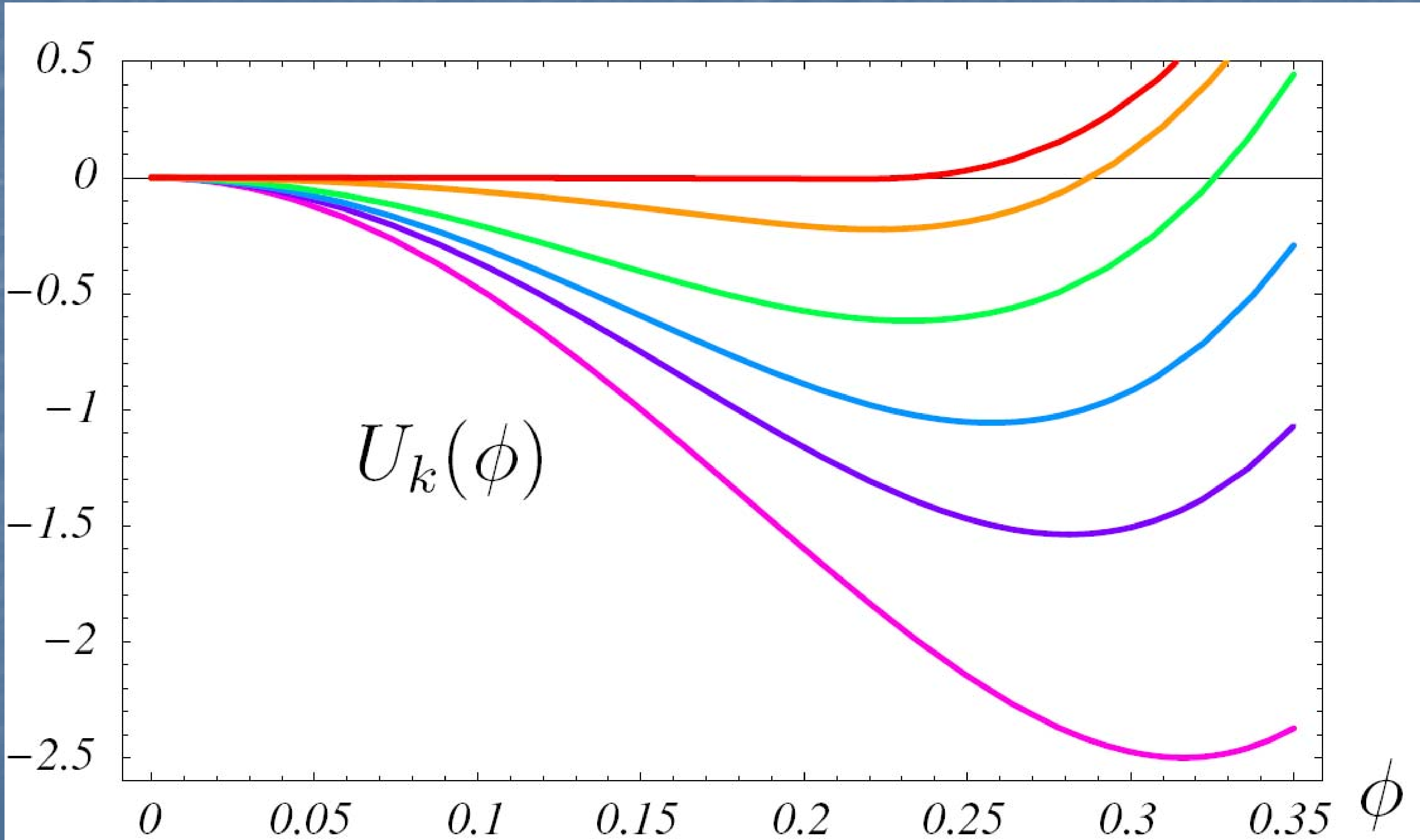
$$\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$$

- The k-dependence of $\Gamma_k[\Phi]$ is determined by a differential functional flow equation which is known in the literature in slightly different forms (see F. J. Wegner and A. Houghton Phys. Rev. A 1973, J. Polchinski Nucl. Phys. B 1984, T. S. Chang et al. Phys. Rep. 1992, C. Wetterich, Nucl. Phys. B 1991, …).
- This gives rise to a class of functionals that interpolate between the classical bare Euclidean action and the full effective action of the theory.
- To evaluate G(p=0), the relevant quantity is the k-dependent effective potential $V_k(\phi)$ which naturally appears in a derivative expansion of $\Gamma_k[\Phi]$ around a space-time constant configuration $\Phi(x) = \phi$

J. Alexandre, V. Branchina and J. Polonyi, Phys. Lett. B 1999



D.Litim, J. M. Pawloski and L. Vergara 2006



M. C. and D. Zappala', Phys. Lett. B 2006

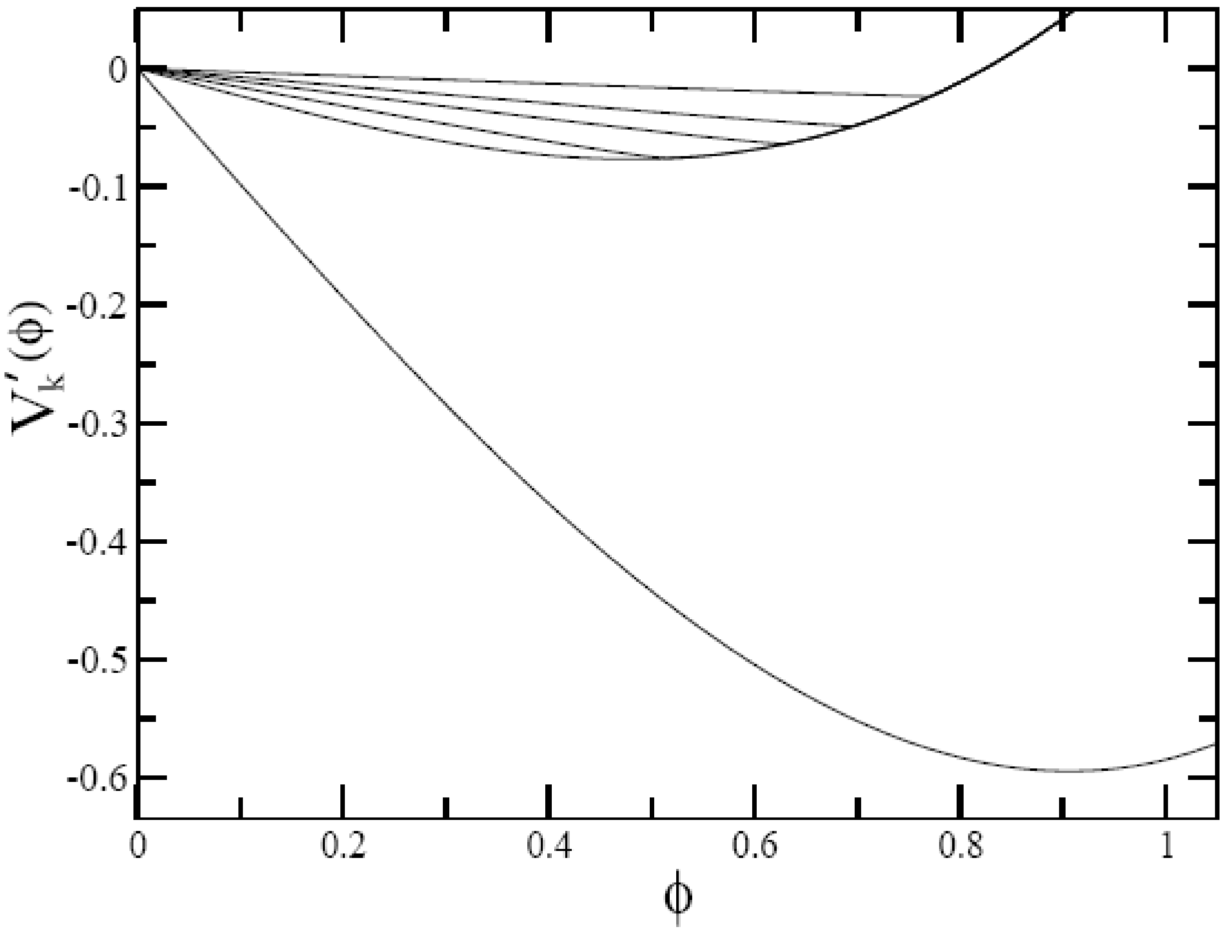


Figure 1: $V'_k(\phi)$ *vs.* ϕ, with $Z = 1$ fixed, at various values of the infrared cutoff k: the lowest curve is for $k = \Lambda = 10$, and then, from bottom to the top, $k = 0.3,\ 0.13,\ 0.1,\ 0.08,\ 0.05$.

- Summarizing: the k-dependent effective potential $V_k(\phi)$, obtained by integrating out shells of quantum modes down to some infrared cutoff k, is clearly approaching convexity in the limit $k \to 0$
- From a physical point of view, this means that convexification is induced by the very long wavelength modes that, so to speak, live in different vacuum states
- Therefore, this well defined theoretical construction supports the identification of $V_{LT}(\varphi)$ (and NOT of $V_{NC}(\varphi)$) as the true effective potential in the infinite-volume limit of the theory
- Explicit calculations of $V'_k(\phi)$ support the conclusion that

$$G^{-1}(p=0) \equiv V''_{k=0}(\pm v)$$

is a two-valued function that includes the solution $G^{-1}(p=0)=0$ as in a massless theory

G(p) from Stevenson's alternative calculation

- Stevenson's problem is to resolve the qualitative conflict (see Coleman and Weinberg) that exists in pure phi^4 theories between 1-loop potential and its RG-improvement. To this end, he starts from the two basic diagrams of the symmetric phase (Mod. Phys. Lett. A 2009)

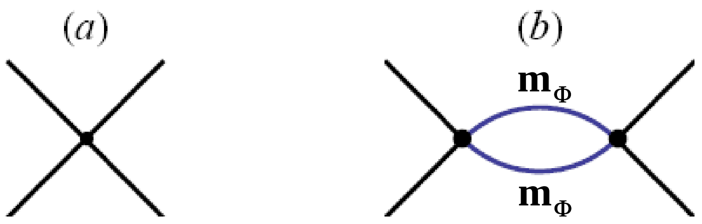


Figure 1: (a) The fundamental interaction. (b) The "fish" diagram, which induces a long-range interaction.

- Diagram (a) gives the repulsive contact potential $+\lambda\delta^{(3)}(\vec{\mathbf{r}})$

- Diagram (b) renormalizes the term $+\lambda\delta^{(3)}(\vec{\mathbf{r}})$ and introduces an attractive tail

$$-\frac{\lambda^2}{\mathbf{r}^3}\mathbf{e}^{-2\mathbf{m}_\Phi \mathbf{r}}$$

that becomes long-range when $\mathbf{m}_\Phi \to \mathbf{0}$ (SSB)

- The existence of two qualitatively different interaction terms suggests to start (in the cutoff theory) from the non-local action

$$S = \int d^4 x \, \frac{1}{2} \partial_\mu \Phi \, \partial^\mu \Phi + \int d^4 x \int d^4 y \; \Phi^2(x) U(x-y) \Phi^2(y),$$

where $$\mathbf{U(x-y) = U_{core}(x-y) + U_{tail}(x-y)}$$

By replacing $$\mathbf{\Phi(x) = \phi + h(x)}$$

the inverse h-field propagator is

$$\mathbf{G^{-1}(x-y) = \left(-\partial_x^2 + 4\tilde{U}(p=0)\phi^2\right)\delta^4(x-y) + 8U(x-y)\phi^2}$$

- In this way (by avoiding double counting of the effects of $\mathbf{U_{core}(x-y)}$ and $\mathbf{U_{tail}(x-y)}$) one can define an alternative RG expansion as in theory with two coupling constants (e.g. as in scalar QED). This analysis eliminates the qualitative conflict between one-loop effective potential and its RG-improved result.

- The tail effect is an infrared effect therefore, as in the standard perturbative treatment of contact interactions, the continuum limit is “trivial”. This means that the propagator becomes free-field. However, due to the presence of the infrared tail, there are deviations in an infinitesimal region near p=0 (that vanishes in a strict continuum limit), i.e.

$$G^{-1}(p) \to (p^2 + m_h^2) \qquad p \neq 0$$

$$G^{-1}(p = 0) = \delta^2 \to 0$$

- where

$$\frac{\delta^2}{m_h^2} \approx \frac{1}{\ln(\frac{\Lambda}{\delta})}$$

- Therefore, for a large but finite cutoff, the theory contains also an infinitesimal infrared scale where deviations from massive free-field behavior show up.

Summarizing

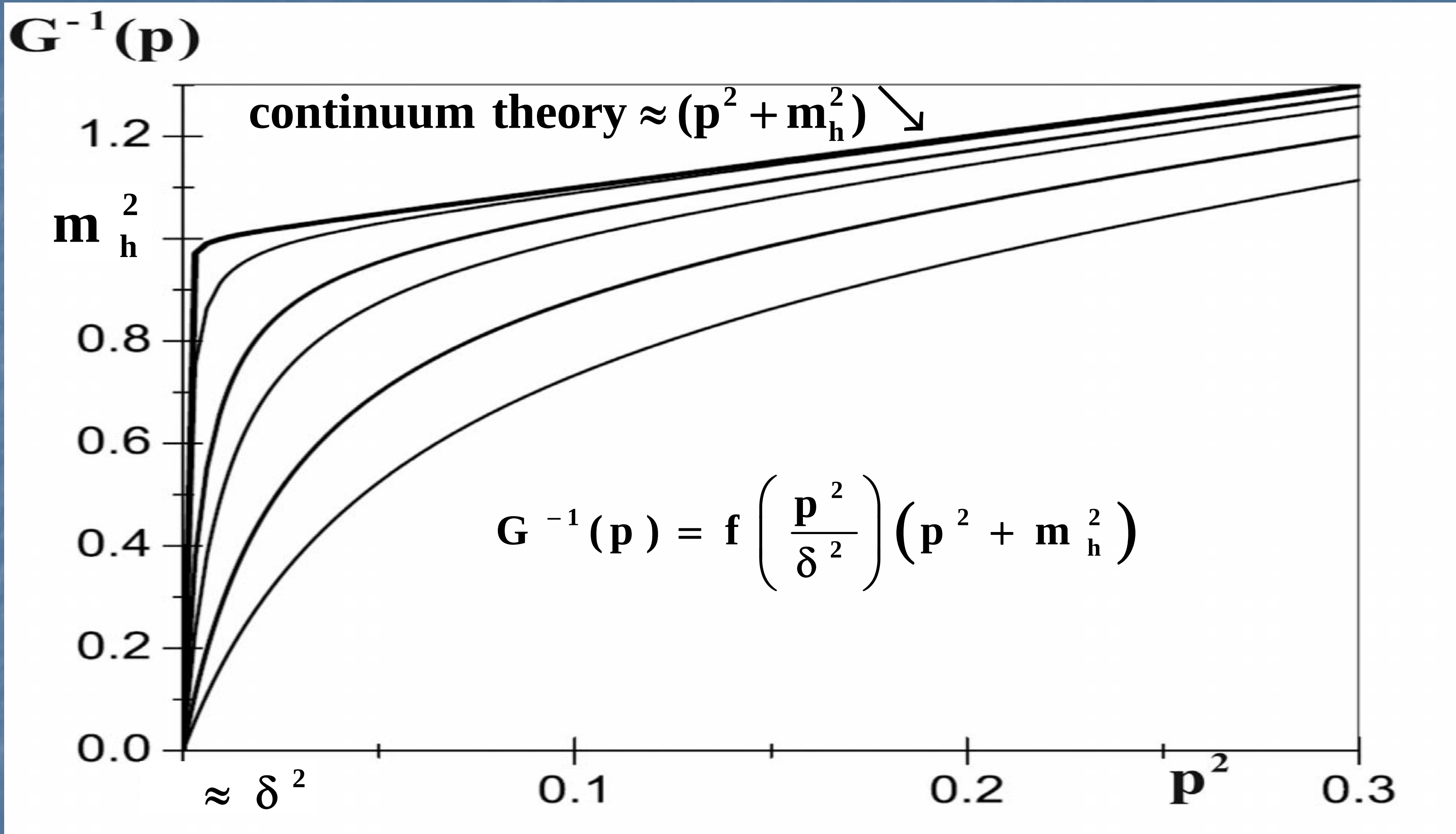
- The general analysis of SSB shows that, beyond the simplest perturbative approximation, $\mathbf{G(p=0)}$ is not so simply related to the Higgs particle mass but is a two-valued function which also includes the solution $\mathbf{G^{-1}(p=0)=0}$ as in massless theory
- Stevenson's analysis shows that, due to the "triviality" of the theory in 4 space-time dimensions, deviations from a free-field behaviour (for the continuum theory) can only occur at p=0, which defines a Lorentz-invariant subset
- For large but finite UV cutoff, the two-valued nature of $\mathbf{G(p=0)}$ suggests that besides the Higgs particle mass, one should also introduce a new "infrared" scale $\delta \ll \mathbf{m_h}$ and consider the form (M.C. PLB 2009)

$$\mathbf{G^{-1}(p)=(p^2+m_h^2)f(\frac{p^2}{\delta^2})}$$

where

$$\mathbf{f\left(\frac{p^2}{\delta^2}\right)\to 1} \quad \text{for} \quad \mathbf{p^2 \gg \delta^2}$$

$$\mathbf{f\left(\frac{p^2}{\delta^2}\right)\to 0} \quad \text{for} \quad \mathbf{p_\mu \to 0}$$



G-1(p)
continuum theory ≈ (p2 + mh2)
1.2
mh2
0.8
0.6
0.4
0.2
0.0
≈ δ2
0.1
0.2
p2
0.3
G−1(p) = f(p2/δ2)(p2 + mh2)

Ultra-weak long range interactions

- A propagator of the form

$$G^{-1}(p) = (p^2 + m_h^2) f\left(\frac{p^2}{\delta^2}\right)$$

- gives an instantaneous potential mediated by

$$D(\vec{r}) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{r}}}{(\vec{p}^2 + m_h^2) f(\vec{p}^2/\delta^2)}$$

- By using the general properties of the Fourier transform this gives an ultra-weak asymptotic 1/r potential

$$\lim_{r\to\infty} D(r) = D_\infty(r) = \frac{\delta^2}{f'(0) m_h^2} \frac{1}{4\pi r}$$

that vanishes in the continuum limit where $\frac{\delta}{m_h} \to 0$

- Consistency with experiments gives a confidence area in the plane (δ, m_h)

Conclusions

- Some numerical and analytic arguments suggest that the standard Higgs boson is NOT a genuine massive field
- The main point is that, beyond perturbation theory, G (p=0) is a two-valued function in the presence of SSB
- However, if the effective scalar self-interaction is "trivial", in the continuum limit the propagator has to become free-field for all but non-zero momenta.
- For large but finite ultraviolet cutoff, from a phenomenological point of view, one expects long-range ultra-weak interactions whose strength should vanish for the continuum theory
- This introduces a new form of infrared-ultraviolet connection with 3 energy scales (Λ, m_h, δ)

  such that $\frac{\delta}{m_h} \to 0$ when $\frac{m_h}{\Lambda} \to 0$
- This should motivate a new generation of numerical simulations on those very large 4D lattices (e.g. 100^4) that are now available with the present computer technology.