Basis independent methods for the Two-Higgs Doublet Model (2HDM)

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Outline

- Motivation for the basis-independent formulation of the 2HDM
- Two approaches to 2HDM studies
- Basis-independent form of the 2HDM Higgs couplings
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- Basis-independent treatment of the Higgs-fermion couplings
- Custodial symmetry in the 2HDM

Pioneering work appeared in papers by Lavoura and Silva in 1994 and by Botella and Silva in 1995. A beautiful exposition of these matters in G.C. Branco, L. Lavoura and J.P. Silva, *CP Violation* (Oxford Univ. Press, 1999) inspired my work. In addition to these authors, I have also been greatly influenced by the work of A. Barroso, W. Bernreuther, P.M. Ferreira, I.F. Ginzberg, B. Grzadkowski, I.P. Ivanov, M. Krawcyzk, E. Ma, M. Maniatis, O. Nachtmann, F. Nagel, C.C. Nishi, P. Osland, M.N. Rebelo, R. Santos, J.A. Silva-Marcos, and A. von Manteuffel.

This talk reviews work that appears in:

1. S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," *Phys. Rev.* **D72**, 035004 (2005).
2. J.F. Gunion, H.E. Haber, "Conditions for CP-violation in the general two-Higgs-doublet model," Phys. Rev. **D72**, 095002 (2005).
3. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan\beta$," *Phys. Rev.* **D74**, 015018 (2006).
4. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. III: CP-violation, custodial symmetry, and the oblique parameters S, T and U," *Phys. Rev.* **D83**, 055017 (2011).
5. P.M. Ferreira, H.E. Haber, M. Maniatis, O. Nachtmann, J.P. Silva, "Geometric picture of generalized-CP and Higgs-family transformations in the two-Higgs-doublet model," Int. J. Mod. Phys. **A26**, 769-808 (2011).

Motivation for the basis-independent formalism

Consider the most general 2HDM potential,

$$\begin{aligned}\mathcal{V} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &\quad + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}\,.\end{aligned}$$

In a general 2HDM, Φ_1 and Φ_2 are indistinguishable fields. A basis change consists of a global U(2) transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Note that the gauge-covariant kinetic energy terms of the scalar fields are invariant with respect to U(2), whereas the scalar potential squared-masses and couplings change under U(2) transformations and thus are *basis-dependent* quantities.

Physical quantities that can be measured in the laboratory must be basis-independent. Thus, any model-independent experimental study of 2HDM phenomena must employ basis-independent methods for analyzing data associated with 2HDM physics.

Caveats

- The most general 2HDM contains large tree-level Higgs-mediated flavor-changing neutral current (FCNC) and CP-violating effects, which are inconsistent with present experimental data over a large range of the 2HDM parameter space. This can be rectified by either
  - fine-tuning of 2HDM parameters to reduce the size of the FCNC and CP-violating effects below the experimentally allowed limits; or
  - imposing additional symmetries (discrete and/or continuous) to eliminate tree-level Higgs-mediated FCNCs and CP-violation. The latter can distinguish between Φ_1 and Φ_2, in which case a choice of basis acquires physical significance.
- Even if additional symmetries are present, they are typically broken, in which case the effective low-energy 2HDM will contain all possible terms consistent with gauge invariance. In this case, the most general 2HDM applies and basis-independent methods are again required.
- Ideally, one would like to use a model-independent analysis to determine which additional symmetries (broken and unbroken) are present. Here again, the basis-independent methods provide a powerful framework for the experimental studies of 2HDM phenomena.

Two approaches to 2HDM studies

1. Work directly with the 2HDM scalar fields

The scalar potential can be rewritten in U(2)-covariant notation:

$$\boxed{\mathcal{V} = Y_{a\bar{b}}\Phi_{\bar{a}}^{\dagger}\Phi_b + \tfrac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^{\dagger}\Phi_b)(\Phi_{\bar{c}}^{\dagger}\Phi_d)}$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \to U_{a\bar{c}}Y_{c\bar{d}}U^{\dagger}_{d\bar{b}}$ and $Z_{a\bar{b}c\bar{d}} \to U_{a\bar{e}}U^{\dagger}_{f\bar{b}}U_{c\bar{g}}U^{\dagger}_{h\bar{d}}Z_{e\bar{f}g\bar{h}}$.

The vacuum expectation values of the two Higgs fields can be parametrized as

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \widehat{v}_a \end{pmatrix}, \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\begin{pmatrix} c_\beta \\ s_\beta\, e^{i\xi} \end{pmatrix},$$

where $v = 246$ GeV and η is arbitrary. Consider the hermitian matrix $V_{a\bar{b}} \equiv \hat{v}_a\hat{v}^*_{\bar{b}}$ with orthonormal eigenvectors $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$. Under a U(2) transformation,

$$\hat{v}_a \to U_{a\bar{b}}\hat{v}_b\,, \qquad \widehat{w}_a \to e^{-i\chi}\,U_{a\bar{b}}\,\widehat{w}_b\,, \qquad \text{where } e^{i\chi} \equiv \det U\,.$$

That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}} \equiv \hat{w}_a \hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$. Moreover $\tan\beta \equiv s_\beta/c_\beta$ is basis-dependent, and hence is *not* in general a physical parameter.

All 2HDM observables must be invariant under a basis transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$. Examples of invariants (which must be real) and potentially complex pseudo-invariants:

$$Y_1 \equiv \mathrm{Tr}\,(YV)\,, \qquad Y_2 \equiv \mathrm{Tr}\,(YW)\,, \qquad Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,,$$

$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,, \qquad Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,,$$

$$Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}} \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{v}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{w}^*_{\bar{c}}\,\widehat{w}_d\,.$$

The pseudo-invariants above transform as

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5\,.$$

Physical quantities must be invariants. For example, the charged Higgs boson mass is $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$. The potential minimum conditions, $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$, are covariant conditions with respect to U(2). Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

The Higgs basis

Define new Higgs doublet fields:

$$H_1 = (H_1^+\,,\; H_1^0) \equiv \widehat{v}_{\bar{a}}^*\Phi_a\,, \qquad\qquad H_2 = (H_2^+\,,\; H_2^0) \equiv \widehat{w}_{\bar{a}}^*\Phi_a\,.$$

Equivalently, $\Phi_a = H_1\hat{v}_a + H_2\hat{w}_a$. It follows that

$$\langle H_1^0\rangle = \frac{v}{\sqrt{2}}\,, \qquad\qquad \langle H_2^0\rangle = 0\,.$$

Under a U(2) transformation, H_1 is *invariant*, whereas $H_2 \to e^{i\chi}H_2$. That is, the Higgs basis is define uniquely up to a possible rephasing of H_2.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 \\ & +\tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \left[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\right]H_1^\dagger H_2 + \text{h.c.}\right\}\,,\end{aligned}$$

which provides an interpretation for the (pseudo-)invariants Y_1, Y_2 Y_3, Z_1, $Z_2, \ldots, Z_7$.

2. Work with gauge-invariant Higgs field bilinears:

One can define SU(2)$_L\times$U(1)$_Y$ gauge invariant scalar field bilinears:

$$K_0=\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2, \qquad K_1=\Phi_1^\dagger\Phi_2+\Phi_2^\dagger\Phi_1,$$
$$K_2=i\Phi_2^\dagger\Phi_1-i\Phi_1^\dagger\Phi_2, \qquad K_3=\Phi_1^\dagger\Phi_1-\Phi_2^\dagger\Phi_2,$$

and define the row vector $\tilde{\boldsymbol{K}}^{\mathsf{T}}=(K_0\,,\ \boldsymbol{K})$. The scalar potential can be rewritten as:

$$V=\tilde{\boldsymbol{K}}^{\mathsf{T}}\tilde{\boldsymbol{\xi}}+\tilde{\boldsymbol{K}}^{\mathsf{T}}\widetilde{E}\,\tilde{\boldsymbol{K}}$$

in terms of a real column vector $\tilde{\boldsymbol{\xi}}$ of squared-mass parameters and a real symmetric 4×4 matrix $\widetilde{E}$ of coupling parameters,

$$\tilde{\boldsymbol{\xi}}=\begin{pmatrix}\xi_0\\ \boldsymbol{\xi}\end{pmatrix}, \qquad \widetilde{E}=\begin{pmatrix}\eta_{00} & \boldsymbol{\eta}^{\mathsf{T}}\\ \boldsymbol{\eta} & E\end{pmatrix}.$$

Under a basis transformation, K_0, ξ_0 and η_{00} are invariant, whereas $\{\boldsymbol{K}\,,\ \boldsymbol{\xi}\,,\ \boldsymbol{\eta}\}$ and E transform as three-dimensional Cartesian vectors and a second-rank tensor, respectively:

$$\{\boldsymbol{K}\,,\ \boldsymbol{\xi}\,,\ \boldsymbol{\eta}\}\to R(U)\{\boldsymbol{K}\,,\ \boldsymbol{\xi}\,,\ \boldsymbol{\eta}\}\,, \qquad E\to R(U)ER(U)^{\mathsf{T}}\,,$$

where $R_{ab}(U)\equiv\frac{1}{2}\mathrm{Tr}\ (U^\dagger\sigma_a U\sigma_b)\in$ SO(3). Basis independent quantities are easily obtained.

Applications of basis-independent methods

- Extracting basis-invariant couplings from 2HDM observables

- Existence of additional symmetries of the 2HDM scalar potential

- CP-violating effects in the Higgs potential
  - CP-transformation properties of neutral Higgs states
  - Potential CP-violation in the Higgs self-interactions
  - Distinguishing between explicit and spontaneous CP-violation

- CP-violating effects in neutral Higgs–fermion interactions

- New sources of custodial symmetry breaking

The Higgs mass-eigenstate basis

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in the Higgs basis The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. Under a U(2) transformation,

$$\theta_{12}\,,\ \theta_{13} \text{ are invariant, and } \quad e^{i\theta_{23}} \to (\det U)^{-1} e^{i\theta_{23}}\,.$$

One can express the mass eigenstate neutral Higgs directly in terms of the original shifted neutral fields, $\overline{\Phi}^0_a \equiv \Phi^0_a - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}}\left[\overline{\Phi}^{0\,\dagger}_{\bar{a}}(q_{k1}\widehat{v}_a + q_{k2}\widehat{w}_a e^{-i\theta_{23}}) + (q^*_{k1}\widehat{v}^*_{\bar{a}} + q^*_{k2}\widehat{w}^*_{\bar{a}} e^{i\theta_{23}})\overline{\Phi}^0_a\right]\,,$$

for $k = 1, \ldots, 4$, where $h_4 = G^0$.

The *invariant* quantities $q_{k\ell}$ are given by:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

The $q_{k\ell}$ are functions of the angles θ_{12} and θ_{13}, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the neutral mass-eigenstate fields are indeed invariant under basis transformations.* Inverting the previous result yields:

$$\Phi_a = \begin{pmatrix} G^+\widehat{v}_a + H^+\widehat{w}_a \\ \dfrac{v}{\sqrt{2}}\widehat{v}_a + \dfrac{1}{\sqrt{2}}\displaystyle\sum_{k=1}^{4}\left(q_{k1}\widehat{v}_a + q_{k2}e^{-i\theta_{23}}\widehat{w}_a\right)h_k \end{pmatrix}.$$

*Likewise, $e^{i\theta_{23}}H^+$ and its charge conjugate are U(2)-invariant fields.

The gauge boson–Higgs boson interactions

$$\mathscr{L}_{VVH} = \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+)$$
$$- g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,,$$

$$\mathscr{L}_{VVHH} = \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k$$
$$+ \left[\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-)$$
$$+ \left\{\left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \mathrm{h.c.}\right\},$$

$$\mathscr{L}_{VHH} = \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu h_k \right] + \mathrm{h.c.} \right\}$$
$$+ \left[i e A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu \right] (G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-)\,.$$

The cubic and quartic Higgs couplings

$$\begin{aligned}
\mathscr{L}_{3h} = -\tfrac{1}{2}v\, h_j h_k h_\ell \Big[& q_{j1}q^*_{k1}\mathrm{Re}(q_{\ell 1})Z_1 + q_{j2}q^*_{k2}\,\mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}Z_5\, e^{-2i\theta_{23}}) \\
& +\mathrm{Re}\Big([2q_{j1}+q^*_{j1}]q^*_{k1}q_{\ell 2}Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2}q_{k2}q_{\ell 2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -v\, h_k G^+G^-\Big[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_6)\Big] + v\, h_k H^+H^-\Big[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_7)\Big] \\
& -\tfrac{1}{2}v\, h_k\Big\{G^-H^+\, e^{i\theta_{23}}\,\Big[q^*_{k2}Z_4 + q_{k2}\, e^{-2i\theta_{23}}Z_5 + 2\mathrm{Re}(q_{k1})Z_6\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}\,,
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_{4h} = -\tfrac{1}{8}h_j h_k h_l h_m \Big[& q_{j1}q_{k1}q^*_{\ell 1}q^*_{m1}Z_1 + q_{j2}q_{k2}q^*_{\ell 2}q^*_{m2}Z_2 + 2q_{j1}q^*_{k1}q_{\ell 2}q^*_{m2}(Z_3+Z_4) \\
& +2\mathrm{Re}(q^*_{j1}q^*_{k1}q_{\ell 2}q_{m2}Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1}q^*_{k1}q^*_{\ell 1}q_{m2}Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}q^*_{m2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2}h_j h_k G^+G^-\Big[q_{j1}q^*_{k1}Z_1 + q_{j2}q^*_{k2}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_6\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2}h_j h_k H^+H^-\Big[q_{j2}q^*_{k2}Z_2 + q_{j1}q^*_{k1}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_7\, e^{-i\theta_{23}})\Big] \\
& -\tfrac{1}{2}h_j h_k\Big\{G^-H^+\, e^{i\theta_{23}}\,\Big[q_{j1}q^*_{k2}Z_4 + q^*_{j1}q_{k2}Z_5\, e^{-2i\theta_{23}} + q_{j1}q^*_{k1}Z_6\, e^{-i\theta_{23}} + q_{j2}q^*_{k2}Z_7\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\} \\
& -\tfrac{1}{2}Z_1G^+G^-G^+G^- - \tfrac{1}{2}Z_2H^+H^-H^+H^- - (Z_3+Z_4)G^+G^-H^+H^- \\
& -\tfrac{1}{2}(Z_5H^+H^+G^-G^- + Z^*_5H^-H^-G^+G^+) - G^+G^-(Z_6H^+G^- + Z^*_6H^-G^+) - H^+H^-(Z_7H^+G^- + Z^*_7H^-G^+)\,.
\end{aligned}$$

Possible symmetries of the 2HDM

Symmetries are often imposed on the general 2HDM to satisfy various phenomenological requirements. This requires a basis-independent catalog of all possible allowed symmetries, first achieved by I.P. Ivanov.

designation	Higgs flavor symmetry group	maximal symmetry group
$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \otimes \mathbb{Z}_2$
Peccei-Quinn	U(1)	O(2)
SO(3)	SO(3)	O(3)
CP1	—	$\mathbb{Z}_2$
CP2	$\mathbb{Z}_2 \otimes \mathbb{Z}_2$	$\mathbb{Z}_2 \otimes \mathbb{Z}_2 \otimes \mathbb{Z}_2$
CP3	O(2)	O(2)$\otimes\mathbb{Z}_2$

The table above lists the possible Higgs flavor groups and corresponding maximal symmetry groups [orthogonal to the global U(1)$_Y$ hypercharge] of the scalar sector of the 2HDM.[†] CP1 is the standard CP symmetry, while CP2 and CP3 are *generalized* CP symmetries, which take the form $\Phi_a \to X_{ab}\Phi_b^*$, for unitary X that is also symmetric, antisymmetric or neither for CP1, CP2 and CP3, respectively.

[†] P.M. Ferreira, H.E. Haber and J.P. Silva, Phys. Rev. D79 (2009) 116004; D82 (2010) 016001.

symmetry class	constraints on $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$	eigenvalues of E
$\mathbb{Z}_2$	$\boldsymbol{\xi} \times \hat{\boldsymbol{e}} = \boldsymbol{\eta} \times \hat{\boldsymbol{e}} = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$	non-degenerate
	or	or
	$\boldsymbol{\xi} \times \hat{\boldsymbol{e}}' = \boldsymbol{\eta} \times \hat{\boldsymbol{e}}' = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$	doubly-degenerate
U(1)	$\boldsymbol{\xi} \times \hat{\boldsymbol{e}} = \boldsymbol{\eta} \times \hat{\boldsymbol{e}} = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$	doubly-degenerate
	or	or
	$\boldsymbol{\xi} \times \boldsymbol{\eta} = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$	triply-degenerate
SO(3)	$(\boldsymbol{\xi}, \boldsymbol{\eta}) = (0,0)$	triply-degenerate
CP1	$\boldsymbol{\xi} \times \boldsymbol{\eta}$ is an eigenvector of E	unconstrained
	or	or
	$\boldsymbol{\xi} \times \boldsymbol{\eta} = 0$; $\boldsymbol{\xi} \cdot \hat{\boldsymbol{e}} = \boldsymbol{\eta} \cdot \hat{\boldsymbol{e}} = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$; neither $\boldsymbol{\xi}$ nor $\boldsymbol{\eta}$ is an eigenvector of E	non-degenerate
	or	or
	$\boldsymbol{\xi} \times \boldsymbol{\eta} = 0$; $\boldsymbol{\xi} \cdot \hat{\boldsymbol{e}}' = \boldsymbol{\eta} \cdot \hat{\boldsymbol{e}}' = 0$; $(\boldsymbol{\xi}, \boldsymbol{\eta}) \neq (0,0)$; neither $\boldsymbol{\xi}$ nor $\boldsymbol{\eta}$ is an eigenvector of E	doubly-degenerate
CP2	$(\boldsymbol{\xi}, \boldsymbol{\eta}) = (0,0)$	non-degenerate
CP3	$(\boldsymbol{\xi}, \boldsymbol{\eta}) = (0,0)$	doubly-degenerate

The symmetry classes and the corresponding constraints on the scalar potential parameters. The unit vector $\hat{\boldsymbol{e}}$ is one of the three eigenvectors of E corresponding to a non-degenerate eigenvalue of E, and the unit vector $\hat{\boldsymbol{e}}'$ is an eigenvector of E corresponding to a doubly-degenerate eigenvalue of E. For further details, see P.M. Ferreira, H.E. Haber, M. Maniatis, O. Nachtmann and J.P. Silva, Int. J. Mod. Phys. A26 (2011) 769. The hierarchy of 2HDM symmetry classes can be exhibited by the following chain:

$$\mathrm{CP1} < \mathbb{Z}_2 < \left\{ \begin{array}{c} \mathrm{U(1)} \\ \mathrm{CP2} \end{array} \right\} < \mathrm{CP3} < \mathrm{SO(3)}\,.$$

Conditions for a CP-conserving Higgs scalar sector

Explicit CP conservation means that the Higgs potential exhibits (at least) a CP1 symmetry. This implies that E, $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ satisfy one of the possible set of constraints listed on the previous table. Equivalently, the necessary and sufficient conditions for an explicitly CP-conserving 2HDM scalar potential consist of the (simultaneous) vanishing of the imaginary parts of four potentially complex invariants:

$$
\begin{aligned}
I_{Y3Z} &\equiv \mathrm{Im}(Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}})\,, \\
I_{2Y2Z} &\equiv \mathrm{Im}(Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}})\,, \\
I_{6Z} &\equiv \mathrm{Im}(Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}})\,, \\
I_{3Y3Z} &\equiv \mathrm{Im}(Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}})\,,
\end{aligned}
$$

where $Z^{(1)}_{a\bar{d}} \equiv \delta_{b\bar{c}} Z_{a\bar{b}c\bar{d}} = Z_{a\bar{b}b\bar{d}}$. If these four invariants vanish, then there exists a *real basis* in which all scalar potential parameters are real.

If *both* the scalar potential and the Higgs vacuum are CP-conserving, then the invariant conditions are much simpler:

$$\mathrm{Im}\,(Z_5^* Z_6^2) = \mathrm{Im}\,(Z_5^* Z_7^2) = \mathrm{Im}\,(Z_6^* Z_7) = 0\,,$$

which are equivalent to conditions first obtained by Lavoura and Silva and by Botella and Silva. In this case a *real Higgs basis* exists where Y_3, Z_5, Z_6 and Z_7 are real (as the scalar potential minimum condition fixes $Y_3 = -\frac{1}{2}Z_6 v^2$). Moreover, the interactions of the Higgs bosons with themselves and with the gauge bosons are be CP-conserving, and the neutral Higgs bosons are eigenstates of CP (with CP-even h^0 and H^0 and CP-odd A^0).

The coefficients of the scalar potential in the Higgs basis (Y_1, Y_2 Y_3, Z_1, $Z_2, \ldots, Z_7$) can be used to evaluate I_{Y3Z}, I_{2Y2Z}, I_{6Z} and I_{3Y3Z}. If these quantities are all real but $\mathrm{Im}\,(Z_5^* Z_6^2) = \mathrm{Im}\,(Z_5^* Z_7^2) = \mathrm{Im}\,(Z_6^* Z_7) = 0$ is *not* satisfied, then CP is spontaneously broken. Namely, no real basis exists where the Higgs vacuum expectation values are both real.

The Higgs-fermion Yukawa couplings

The Yukawa Lagrangian, in terms of the quark mass-eigenstate fields, is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \widetilde{\Phi}^0_{\bar a} \eta^U_a U_R + \overline{D}_L K^\dagger \widetilde{\Phi}^-_{\bar a} \eta^U_a U_R + \overline{U}_L K \Phi^+_a \eta^{D\,\dagger}_{\bar a} D_R + \overline{D}_L \Phi^0_a \eta^{D\,\dagger}_{\bar a} D_R + {\rm h.c.}\,,$$

where $\widetilde{\Phi}_{\bar a} \equiv (\widetilde{\Phi}^0\,,\,\widetilde{\Phi}^-) = i\sigma_2 \Phi^*_{\bar a}$ and K is the CKM mixing matrix. The $\eta^{U,D}$ are 3×3 Yukawa coupling matrices. It is convenient to write:

$$\eta^Q_a = \kappa^Q \widehat{v}_a + \rho^Q \widehat{w}_a \qquad \Longrightarrow \qquad \kappa^Q \equiv \widehat{v}^*_{\bar a} \eta^Q_a \quad \text{and} \quad \rho^Q \equiv \widehat{w}^*_{\bar a} \eta^Q_a\,, \qquad (Q = U \text{ or } D)\,.$$

Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det U)\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively, whereas the matrices ρ^U and ρ^D are independent complex 3×3 matrices. In particular,

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = {\rm diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}}\kappa^{D\,\dagger} = {\rm diag}(m_d\,,\, m_s\,,\, m_b)\,.$$

The fermion–Higgs boson interactions

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is [with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$]:

$$
\begin{aligned}
-\mathscr{L}_Y = &\frac{1}{v}\overline{D}\left\{M_D(q_{k1}P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q_{k2}^*\, e^{i\theta_{23}}\rho^D P_L\right]\right\}Dh_k \\
&+\frac{1}{v}\overline{U}\left\{M_U(q_{k1}P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}}\left[q_{k2}^*\, e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\right\}Uh_k \\
&+\left\{\overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right]DH^+ + \frac{\sqrt{2}}{v}\overline{U}\left[KM_D P_R - M_U K P_L\right]DG^+ + \text{h.c.}\right\}
\end{aligned}
$$

Since $e^{i\theta_{23}}H^+$ and the h_k are invariant fields, $\mathscr{L}_Y$ depends only on invariant quantities: the matrices M_Q and $\rho^Q e^{i\theta_{23}}$ and the invariant angles θ_{12} and θ_{13}. The unphysical parameter $\tan\beta$ does *not* appear.

The couplings of the neutral Higgs bosons to quark pairs are generically CP-violating due to the fact that the q_{k2} and the matrices $e^{i\theta_{23}}\rho^Q$ are not generally either pure real or pure imaginary.

Additional symmetries in the Higgs-fermion interactions

A general 2HDM exhibits CP-violating neutral Higgs boson couplings to fermions and tree-level FCNCs mediated by neutral Higgs boson exchanges. These effects can be removed by symmetry. Once again, a basis-independent formulation of such symmetries are useful (as these could be extracted in principle from experimental data).

- Condition for CP-conserving neutral Higgs–fermion interactions:

$$Z_5(\rho^Q)^2\,,\ Z_6\rho^Q \ \text{ and }\ Z_7\rho^Q \ \text{ are real matrices } (Q = U,\ D \text{ and } E).$$

  Remark: CP symmetry cannot be exact due to the unremovable phase in the CKM matrix that enters via the charged current interactions mediated by either $W^\pm$, $H^\pm$ or $G^\pm$ exchange.

- Type-I and Type-II Higgs-fermion interactions

$$\begin{aligned} \epsilon_{\bar{a}\bar{b}}\eta_a^D\eta_b^U = \epsilon_{ab}\eta_{\bar{a}}^{D\,\dagger}\eta_{\bar{b}}^{U\,\dagger} &= 0\,, \qquad \text{type-I}\,, \\ \delta_{a\bar{b}}\,\eta_{\bar{a}}^{D\,\dagger}\eta_b^U &= 0\,, \qquad \text{type-II}\,, \end{aligned}$$

which can be implemented with a $\mathbb{Z}_2$ symmetry (with appropriate choices for the transformations of the scalar and fermion fields). In both cases $\rho^Q \propto M_Q$ (for $Q = U$, D). Hence, there are no off-diagonal neutral Higgs–fermion couplings. The Type-I and Type-II conditions can be implemented via discrete symmetries (or supersymmetry), which distinguish the two (otherwise identical) Higgs doublets.

For example, in a Type-II model, $\tan\beta$ is the ratio of the neutral Higgs vacuum expectation values in the special basis in which $\eta_1^U = \eta_2^D = 0$. Indeed, $\tan\beta$ has been promoted to a physical (invariant) parameter,

$$\tan\beta = \frac{v}{3\sqrt{2}}\left|\mathrm{Tr}\ \left(\rho^D M_D^{-1}\right)\right|\,.$$

Custodial symmetry in the 2HDM

In the Standard model, the scalar sector exhibits a global SU(2)$_L\times$SU(2)$_R$ symmetry that is violated only by hypercharge gauge interactions and the Higgs-fermion Yukawa couplings. In the custodial symmetric limit the electroweak ρ-parameter,

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos\theta_W} = 1\,,$$

to all orders in perturbation theory. Including the breaking effects generate finite radiative corrections to $\rho = 1$.

Pomarol and Vega studied the implications of custodial symmetry for the 2HDM in 1994. They identified two separate realizations, but failed to realize that their two cases were actually related by a change of Higgs basis (also noted recently by B. Grzadkowski, M. Maniatis and J. Wudka). Clearly, basis-independent methods can be valuable here.

Define the 2×2 matrices $\mathbb{M}_1$ and $\mathbb{M}_2$, with columns made up of Higgs-basis fields,

$$\mathbb{M}_1 \equiv [i\sigma_2 H_1^* \,, \quad H_1] \,, \qquad\qquad \mathbb{M}_2 \equiv \left[i\sigma_2 (e^{i\chi} H_2)^* \,, \quad e^{i\chi} H_2\right] \,,$$

where χ reflects the phase freedom in defining the Higgs basis. Under a global SU(2)$_L \times$SU(2)$_R$ transformation, $\mathbb{M}_i \to L\mathbb{M}_i R^\dagger$ $(i = 1, 2)$, where L, $R \in \mathrm{SU}(2)$. The vacuum preserves the diagonal SU(2) custodial symmetry (corresponding to $L = R$), since $\langle \mathbb{M}_1 \rangle = (v/\sqrt{2})\mathbb{1}_{2\times 2}$ and $\langle \mathbb{M}_2 \rangle = 0$. Then, imposing the SU(2)$_L \times$SU(2)$_R$ symmetry on the scalar potential in the Higgs basis yields:

$$Z_4 = Z_5 e^{-2i\chi} \,, \qquad \mathrm{Im}(Y_3 e^{-i\chi}) = \mathrm{Im}(Z_6 e^{-i\chi}) = \mathrm{Im}(Z_7 e^{-i\chi}) = 0 \,.$$

Because Z_4 is real, it follows that $\mathrm{Im}(Z_5^* Y_3^2) = \mathrm{Im}(Z_5^* Z_6^2) = \mathrm{Im}(Z_5^* Z_7^2) = 0$. That is, custodial symmetry implies that the Higgs scalar potential is CP-conserving.

The corresponding *basis-independent* conditions for custodial symmetry are:

$$Z_4 = \begin{cases} \dfrac{\mathrm{Re}(Z_5^* Z_6^2)}{|Z_6|^2} = \epsilon_{56}|Z_5|\,, & \text{if } Z_6 \neq 0\,, \\ \dfrac{\mathrm{Re}(Z_5^* Z_7^2)}{|Z_7|^2} = \epsilon_{57}|Z_5|\,, & \text{if } Z_7 \neq 0\,, \\ \pm|Z_5|\,, & \text{if } Y_3 = Z_6 = Z_7 = 0\,. \end{cases}$$

In a real Higgs basis where Z_6 or Z_7 is non-zero, $\epsilon_{56} = \epsilon_{57} = \mathrm{sgn}\; Z_5$, in which case custodial symmetry implies that $Z_4 = Z_5$[‡] and $m_{H^\pm} = m_A$.

In contrast, if $Y_3 = Z_6 = Z_7 = 0$, then one is free to transform $H_2 \to iH_2$ so that the sign of Z_5 is not physical. Moreover, the neutral Higgs spectrum consists of one CP-even Higgs boson with Standard Model couplings and two neutral states, h_a and h_b with opposite-sign CP but whose absolute CP quantum numbers cannot be determined from the bosonic sector.

[‡]The sign of Z_5 is invariant under an O(2) transformation between any two real bases.

If we include the effects of the Higgs-fermion interactions, then the absolute CP-quantum numbers of h_a and h_b can be determined. In fact, these states may not be eigenstates of CP due to possible CP-violation in the Yukawa couplings, which can arise even in the custodial limit where

$$M_U = M_D \,, \qquad\qquad (e^{i\theta_{23}}\rho^D)^\dagger = e^{i\theta_{23}}\rho^U \,,$$

Indeed, these conditions do *not* impose CP-conservation on the neutral Higgs-fermion interactions (since the latter requires that $e^{i\theta_{23}}\rho^U$, $e^{i\theta_{23}}\rho^D$ are both either real or pure imaginary matrices).

Thus, for the case of $Y_3 = Z_6 = Z_7 = 0$ with custodial symmetry, $H^\pm$ is mass-degenerate with a neutral scalar that is CP-even, CP-odd or a state of indefinite CP, depending on whether $e^{i\theta_{23}}\rho^Q$ is real, purely imaginary or complex.[§]

[§]The case of $H^\pm$ degenerate in mass with a CP-even scalar was the *twisted* scenario of Gerard and Herquet, although its origin is clearer in the basis-independent approach.

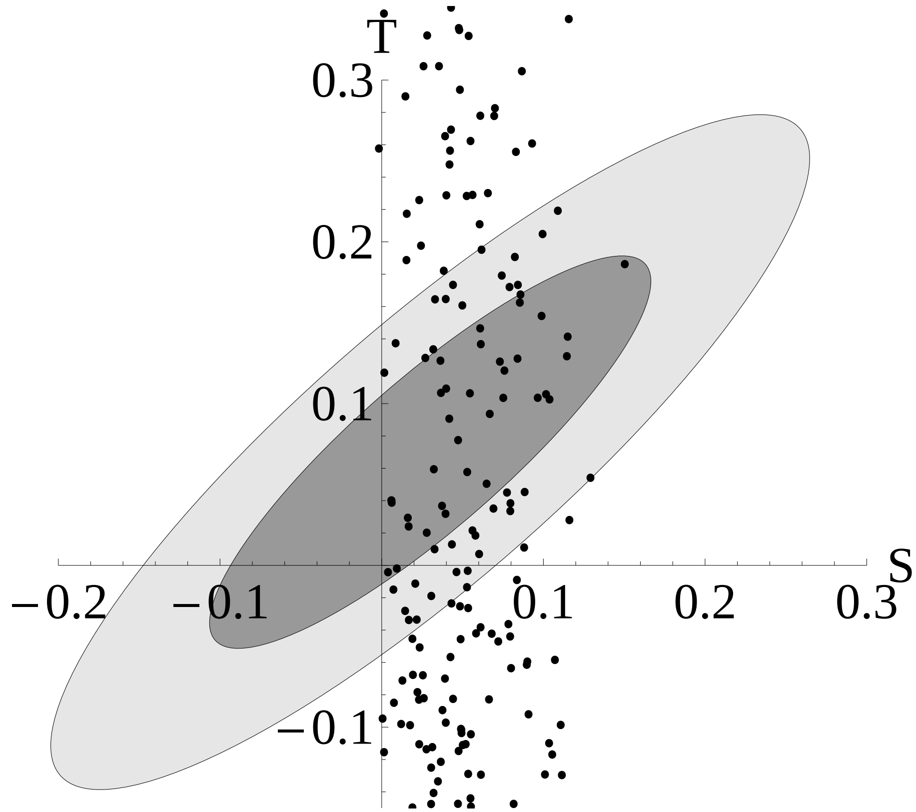
If the custodial symmetry is violated, then one-loop radiative corrections can shift the tree-level result of $\rho = 1$. Denoting $\alpha T \equiv \delta\rho = \rho - 1$, we find that the contribution of a general (possibly CP-violating) Higgs sector to the T parameter is given by the basis independent result:

$$\alpha T = \frac{g^2}{64\pi^2 m_W^2}\left[\sum_{k=1}^{3} |q_{k2}|^2 F(m_k^2, m_{H^\pm}^2) - q_{k1}^2 F(m_i^2, m_j^2)\right] + \mathcal{O}(g'^2)\,, \quad i \neq j \neq k\,,$$

where $m_k \equiv m_{h_k}$ and

$$F(x,y) \equiv \tfrac{1}{2}(x+y) - \frac{xy}{x-y}\ln(x/y)\,, \qquad\qquad F(x,x) = 0\,.$$

This result is consistent with a previous computation of Grimus, Lavoura, Ogreid and Osland. Basis-independent expressions for the S and U parameters have also been obtained by Haber and O'Neil.



- The general 2HDM parameters are constrained mainly by T.
- In the decoupling limit, the lightest Higgs mass is constrained in the same manner as in the SM.
- Away from the decoupling limit, regions exist in which the lightest Higgs boson can be significantly heavier than the SM Higgs boson.
- Away from the decoupling limit, the largest allowed mass-splitting between $H^{\pm}$ and the CP-odd Higgs boson occurs before reaching the unitarity limits of the Higgs couplings.

Lessons for future work

- Basis-independent methods provide a powerful technique for studying the theoretical structure of the two-Higgs doublet model.

- These methods provide insight into the conditions for CP-conservation (and violation), as well as other symmetries of the 2HDM that can distinguish between the two Higgs doublets.

- The basis-independent analysis also clarifies the conditions for custodial symmetry and its breaking.

- It is now possible to perform a completely model-independent scan of the 2HDM parameter space. Constraints on this parameter space due to precision electroweak measurements can be obtained, and provide a possible method for avoiding a Higgs boson mass below 200 GeV.