August 27, 2011
SCALARS2011

Model-independent study of neutral Higgs sector via $t\bar{t}$ productions at muon colliders

Zenrō HIOKI

University of Tokushima

Tokushima 770-8502, Japan

C O N T E N T S

This talk is based on

Collaboration with Kazumasa OHKUMA

1. Introduction

Standard model of EW interaction (SM) :
quite successful but the top and Higgs sectors are still not fully-tested.

If any new physics within our reach, its effects will be likely to appear in those sectors as anomalous interactions.

$$\Downarrow$$

Anomalous top-quark couplings could be tested, e.g., at LHC/ILC.

However it is not easy to study Higgs sector there.

$$\Downarrow$$

Muon colliders: proposed as an ideal machine to explore Higgs properties.

$$\Uparrow$$

Many authors studied direct Higgs productions, and/or $\mu\bar{\mu} \longrightarrow \langle Higgs \rangle \longrightarrow t\bar{t}$ in some specific models with multi Higgs doublets, like MSSM.

As a complementary work to them,
we study possible anomalous neutral-Higgs interactions with $\mu\bar{\mu}$ and $t\bar{t}$ in a model-independent way through $\mu\bar{\mu} \to t\bar{t}$ processes.

Our main purpose:

To study to what extent we could draw a general conclusion on those interactions without assuming any particular models in off-resonance region.

2. Framework

We express the standard-model Higgs as h and the non-standard neutral Higgs as H.

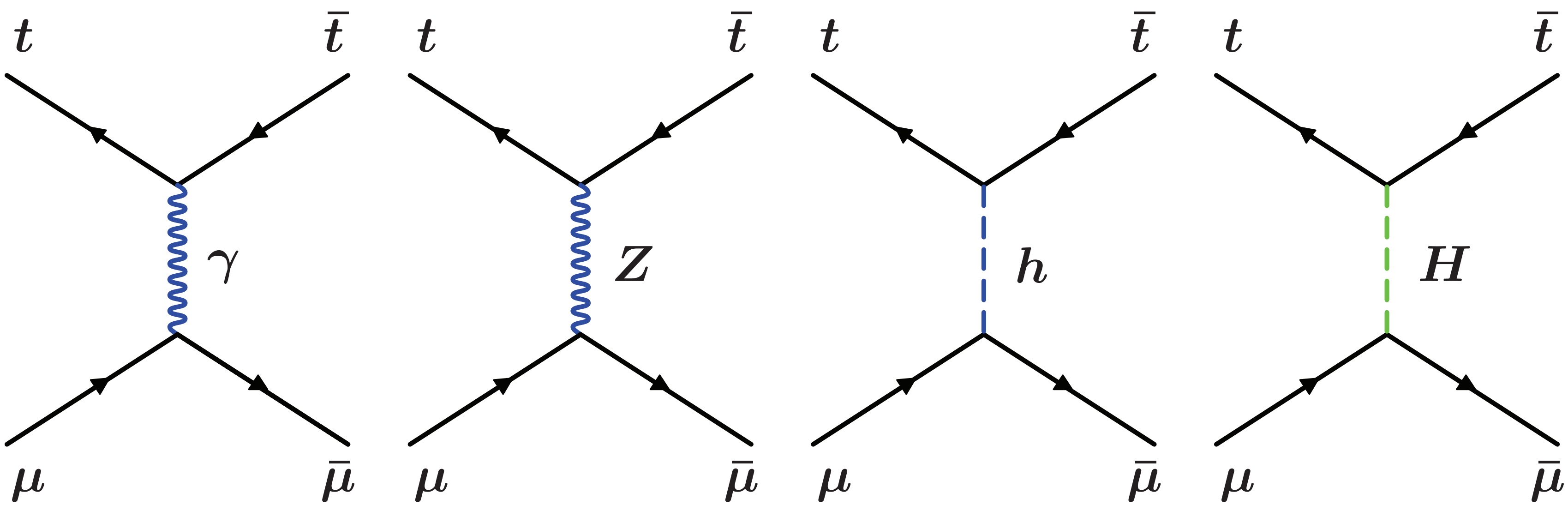

Figure 1: Feynman diagrams for $\mu\bar{\mu} \to (\gamma, Z, h, H) \to t\bar{t}$

Effective amplitude

The invariant amplitude of $\mu\bar{\mu} \to (\gamma, Z, h, H) \to t\bar{t}$ corresponding to Figure 1 is given as follows:

$$\mathcal{M}(\mu\bar{\mu} \to t\bar{t}) = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_h + \mathcal{M}_H, \tag{1}$$

where $\mathcal{M}_{\gamma,Z,h}$ are the standard γ, Z and Higgs exchange terms

$\mathcal{M}_H$ is the non-SM Higgs term in the most general covariant form

$$\mathcal{M}_H = D_H(s)\,\bar{u}(p_t)(a_t + b_t\gamma_5)v(p_{\bar{t}}) \cdot \bar{v}(p_{\bar{\mu}})(a_\mu + b_\mu\gamma_5)u(p_\mu), \tag{2}$$

with

$$D_H(s) \equiv \frac{m_\mu m_t}{v^2}\frac{1}{m_H^2 - s - im_H\Gamma_H}, \tag{3}$$

Γ_H being the total width of H and $v(= 246$ GeV) being the vacuum expectation value of the SM Higgs field.

Note our frame can incorporate any number of Higgs exchange terms:

$$\begin{aligned}
\mathcal{M}[\text{Non-SM Higgs}] &= \sum_{i=1}^{N} D_{H_i}(s)\,\bar{u}(p_t)(a_t^i + b_t^i\gamma_5)v(p_{\bar{t}}) \\
&\qquad\qquad\qquad \times \bar{v}(p_{\bar{\mu}})(a_\mu^i + b_\mu^i\gamma_5)u(p_\mu) \\
&= \sum_i a_t^i a_\mu^i D_{H_i}(s)\,\bar{u}(p_t)v(p_{\bar{t}}) \cdot \bar{v}(p_{\bar{\mu}})u(p_\mu) \\
&+ \sum_i a_t^i b_\mu^i D_{H_i}(s)\,\bar{u}(p_t)v(p_{\bar{t}}) \cdot \bar{v}(p_{\bar{\mu}})\gamma_5 u(p_\mu) \\
&+ \sum_i b_t^i a_\mu^i D_{H_i}(s)\,\bar{u}(p_t)\gamma_5 v(p_{\bar{t}}) \cdot \bar{v}(p_{\bar{\mu}})u(p_\mu) \\
&+ \sum_i b_t^i b_\mu^i D_{H_i}(s)\,\bar{u}(p_t)\gamma_5 v(p_{\bar{t}}) \cdot \bar{v}(p_{\bar{\mu}})\gamma_5 u(p_\mu). \qquad (4)
\end{aligned}$$

Compare One-Higgs and N-Higgs amplitudes (scalar·scalar part, e.g.)

$$a_t a_\mu D_H\,\bar{u}v \cdot \bar{v}u \qquad \Leftrightarrow \qquad \sum_i a_t^i a_\mu^i D_{H_i}\,\bar{u}v \cdot \bar{v}u$$

Thus, all you have to do to get N-Higgs amplitude is replace as

$$a_t a_\mu \Longrightarrow \sum_i a_t^i a_\mu^i D_{H_i}(s)/D_{H_1}(s), \tag{5}$$

$$a_t b_\mu \Longrightarrow \sum_i a_t^i b_\mu^i D_{H_i}(s)/D_{H_1}(s), \tag{6}$$

$$b_t a_\mu \Longrightarrow \sum_i b_t^i a_\mu^i D_{H_i}(s)/D_{H_1}(s), \tag{7}$$

$$b_t b_\mu \Longrightarrow \sum_i b_t^i b_\mu^i D_{H_i}(s)/D_{H_1}(s). \tag{8}$$

Beam polarization

The beam polarization P for spin-component $\pm s$ particles

$$P = (\rho_{+s} - \rho_{-s})/(\rho_{+s} + \rho_{-s}), \tag{9}$$

where $\rho_{\pm s}$ is the number density of the particle in each beam.

The μ and $\bar{\mu}$ spin vectors in the $\mu\bar{\mu}$ CM frame:

$$s^{\alpha} = (P_L\gamma\beta, P_T\cos\phi, P_T\sin\phi, P_L\gamma)\,, \tag{10}$$

$$\bar{s}^{\alpha} = (\bar{P}_L\gamma\beta, \bar{P}_T\cos\bar{\phi}, \bar{P}_T\sin\bar{\phi}, -\bar{P}_L\gamma)\,, \tag{11}$$

(ϕ is the azimuthal angle of s and $P_T = \sqrt{P^2 - P_L^2}$)

3. CP-violating asymmetries

The angular distribution of t:

$$\frac{d}{d\cos\theta}\sigma(\mu\bar{\mu} \to t\bar{t}) = \frac{1}{32\pi s}\frac{|p_t|}{|p_\mu|}|\mathcal{M}(\mu\bar{\mu} \to t\bar{t})|^2. \tag{12}$$

<u>Numerical results</u>

We study two CP-violating asymmetries $A_{L,T}$ for $|P_L| = 1$ or $|P_T| = 1$:

- Longitudinal-polarization asymmetry

$$A_L = \frac{\sigma(++) - \sigma(--)}{\sigma(++) + \sigma(--)}, \tag{13}$$

where $\sigma(\pm\pm)$ express the cross sections for $P_L = \bar{P}_L = \pm 1$.

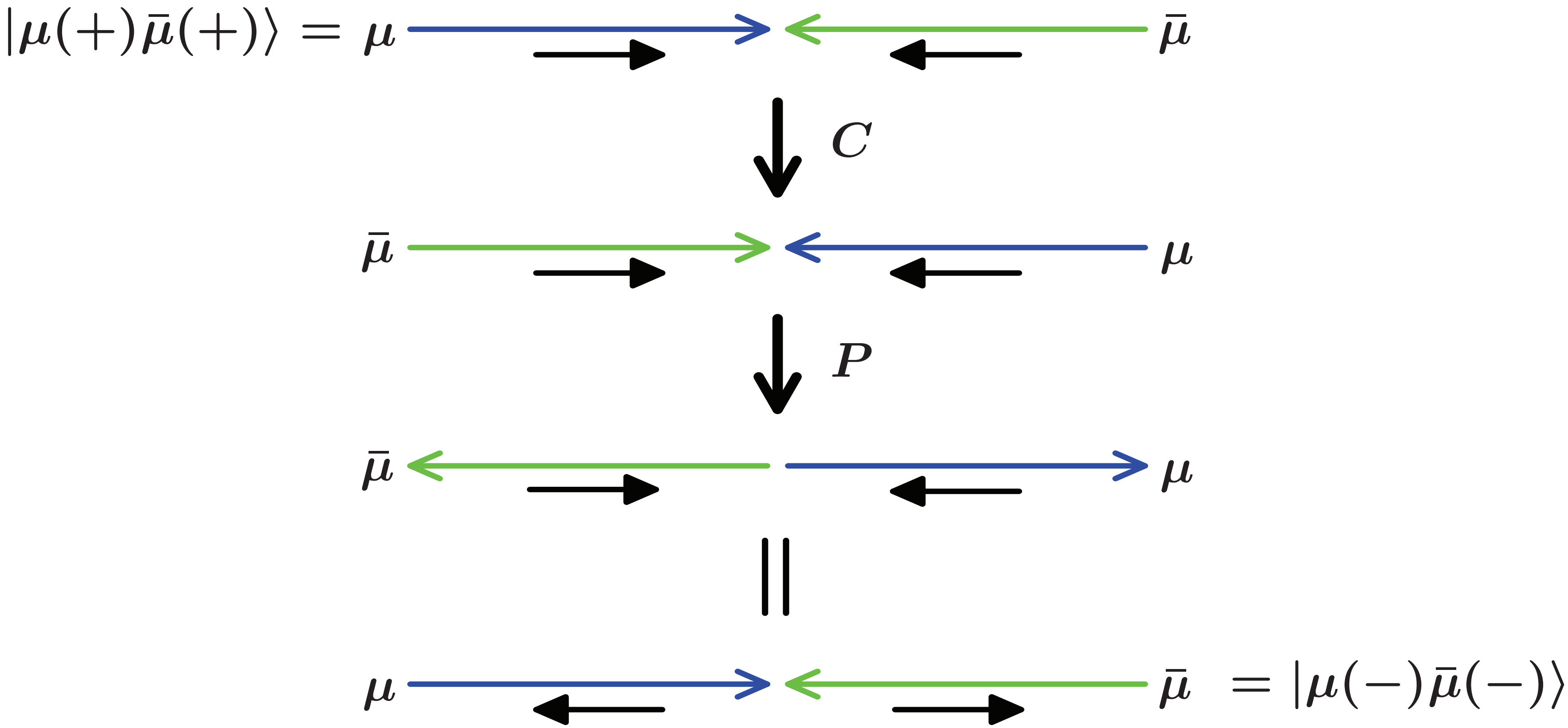

Figure 2: CP **transformation between** $|\mu(\pm)\bar{\mu}(\pm)\rangle$

- **Transverse-polarization asymmetry**

$$A_T = \frac{\sigma(\chi = \pi/2) - \sigma(\chi = -\pi/2)}{\sigma(\chi = \pi/2) + \sigma(\chi = -\pi/2)}, \tag{14}$$

where $\sigma(\chi = \pm\pi/2)$ are the cross sections for $P_T = \bar{P}_T = 1$ with $\chi \equiv \phi - \bar{\phi} = \pm\pi/2$.

Note $|\chi| = \pi/2$ maximizes the CP-violation effects.

The other parameters are taken as

$$\sin^2\theta_W = 0.23, \quad M_Z = 91.187\ \mathrm{GeV}, \quad v = 246\ \mathrm{GeV},$$

$$m_t = 174\ \mathrm{GeV}, \quad m_\mu = 105.658\ \mathrm{MeV}, \quad m_h = 150\ \mathrm{GeV},$$

and we compute Γ_H as $\Gamma_H = \Gamma_h(m_H)$.

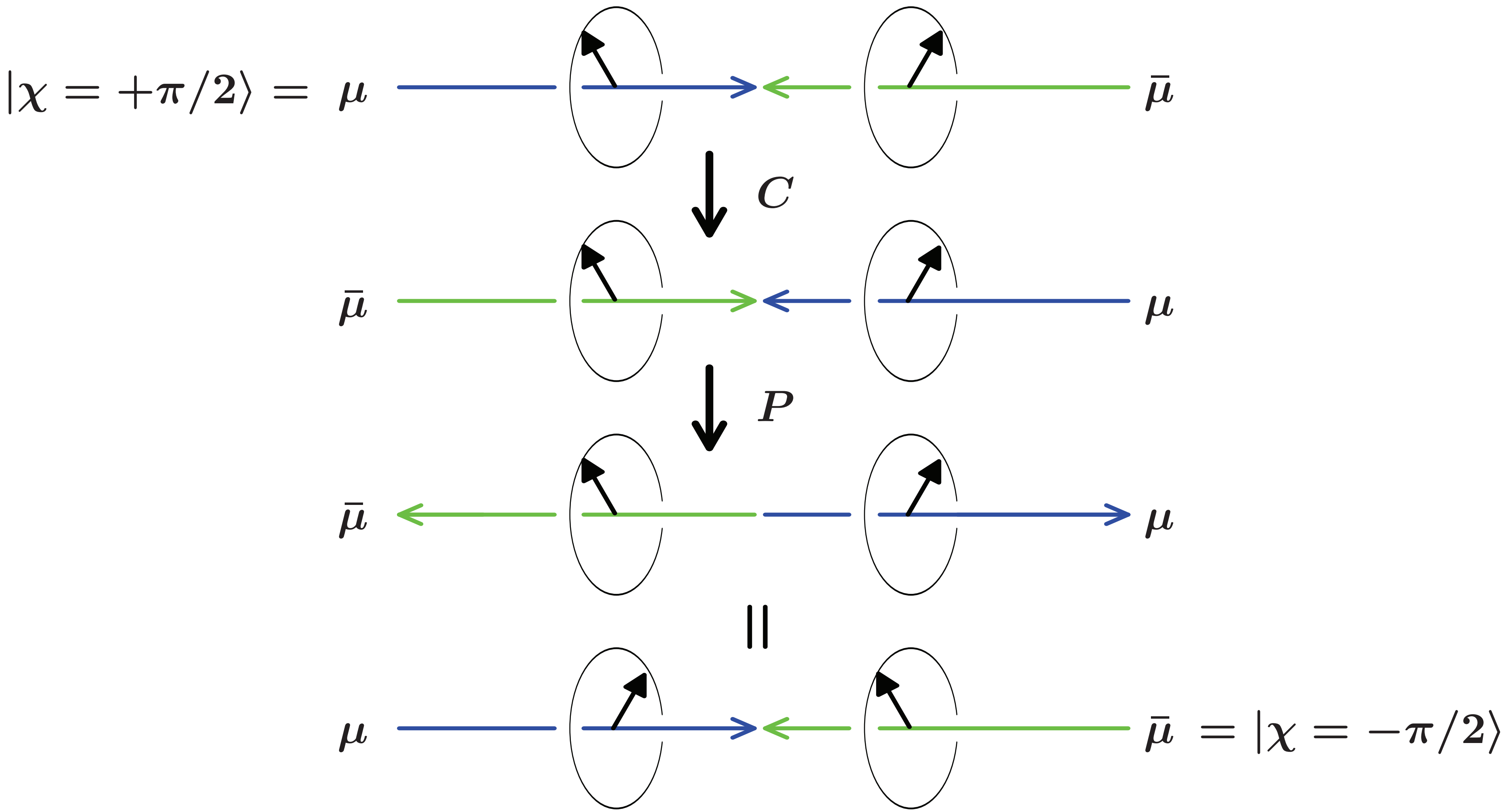

Figure 3: CP **transformation between** $|\chi = \pm\pi/2\rangle$

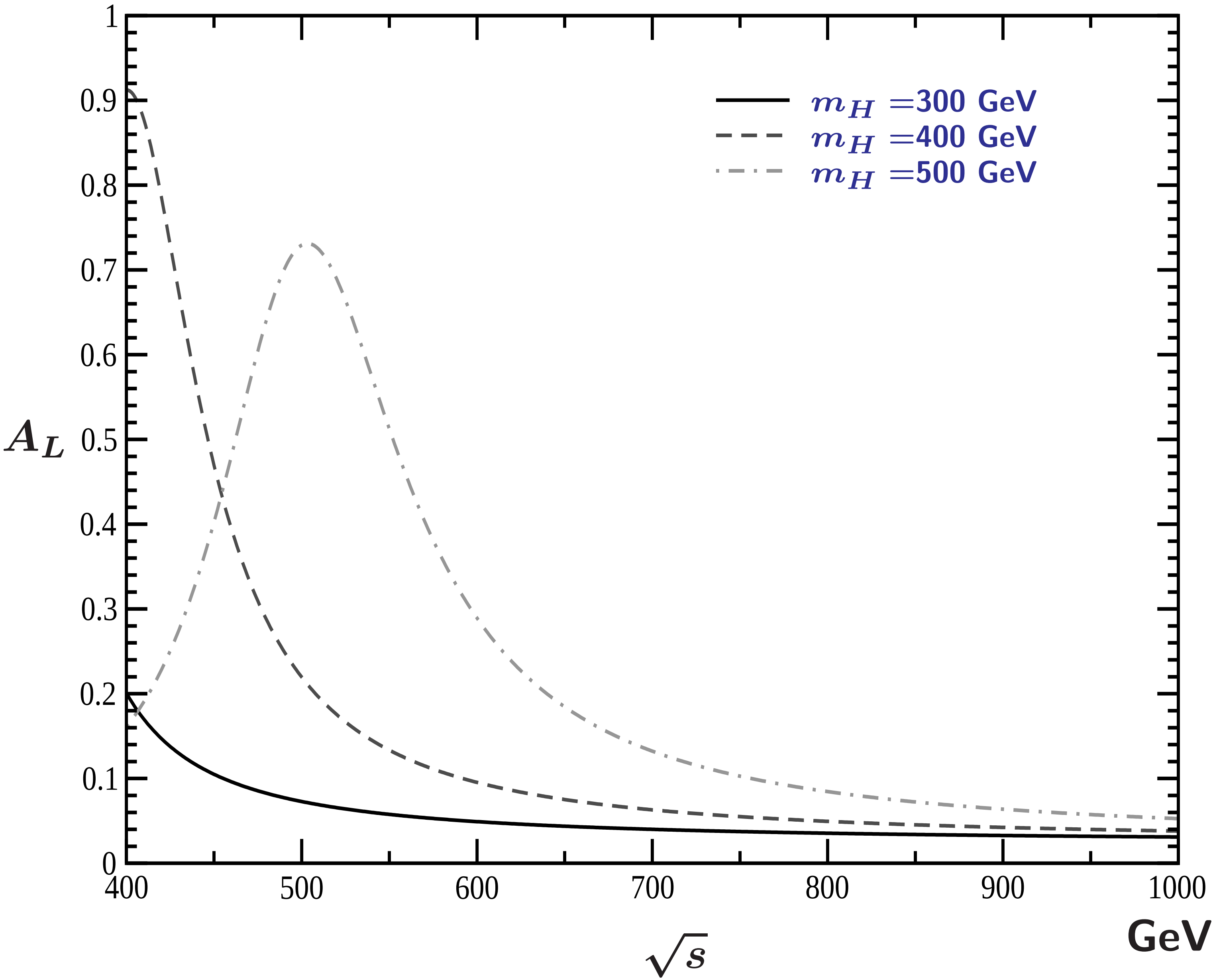

Figure 4: $\sqrt{s}$ **dependence of** A_L **for** $\mathrm{Re}\, a_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, b_{t,\mu} = 0.2$

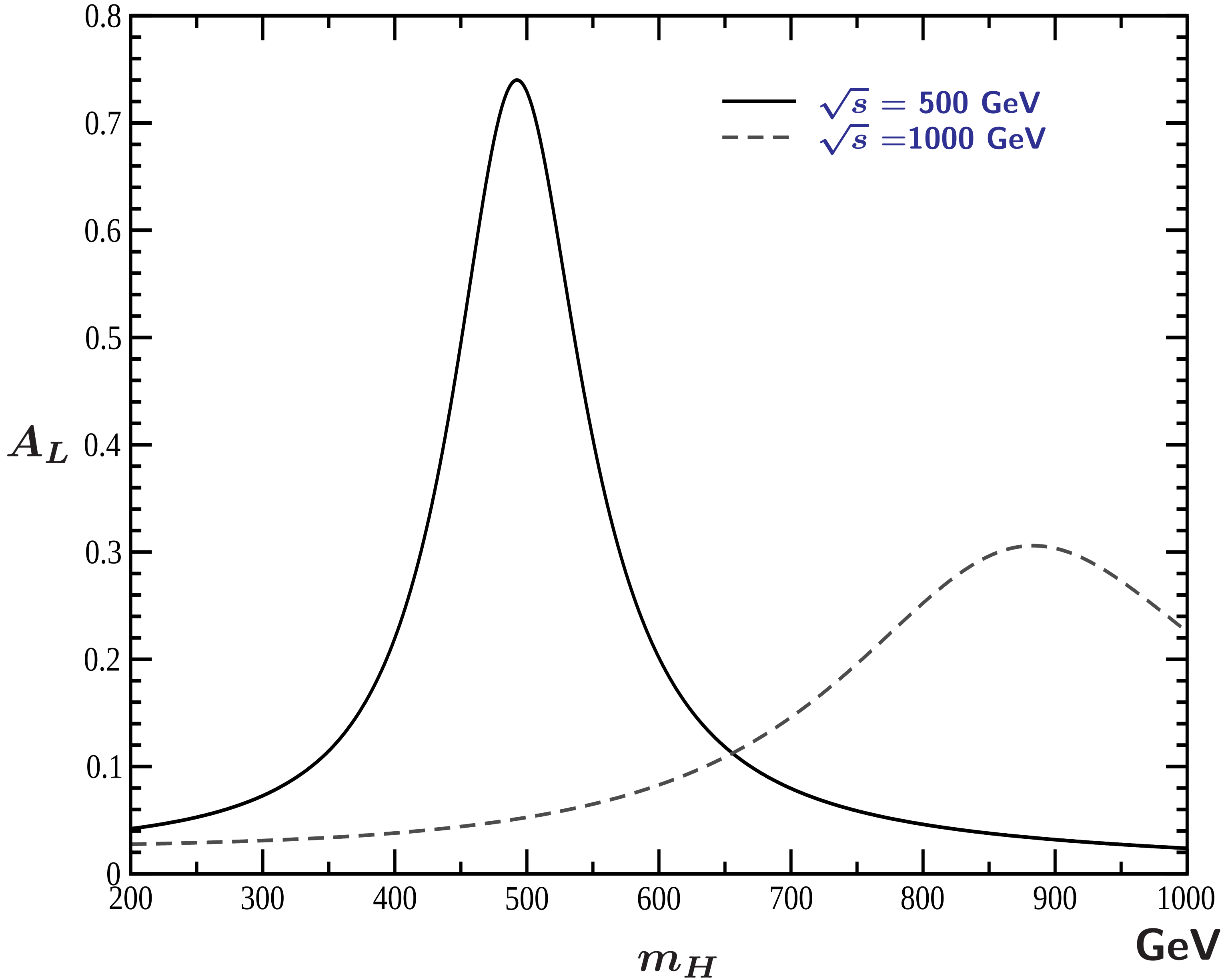

Figure 5: m_H **dependence of** A_L **for** $\mathrm{Re}\, a_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, b_{t,\mu} = 0.2$

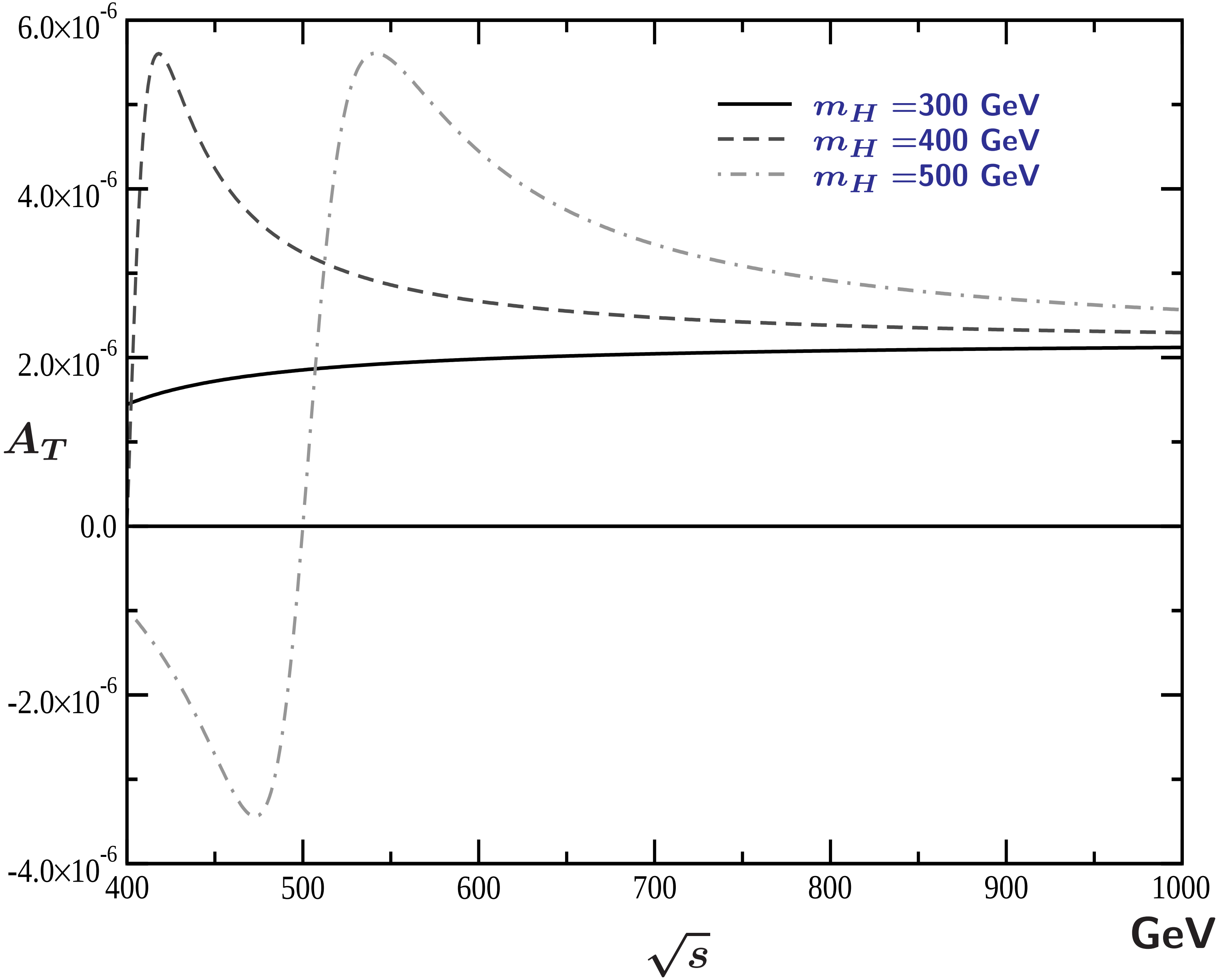

Figure 6: $\sqrt{s}$ **dependence of** A_T **for** $\mathrm{Re}\, a_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, b_{t,\mu} = 0.2$

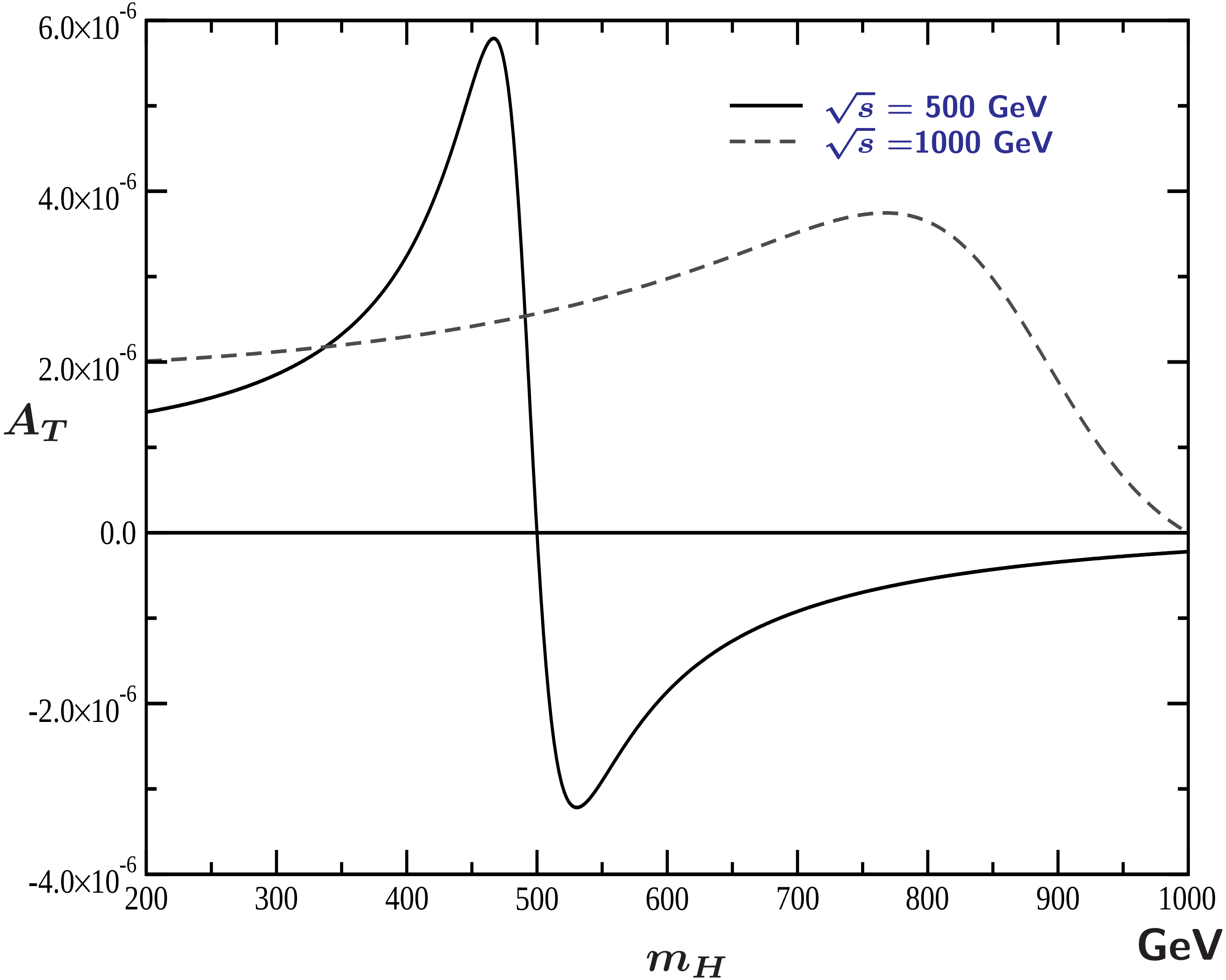

Figure 7: m_H **dependence of** A_T **for** $\mathrm{Re}\, a_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, b_{t,\mu} = 0.2$

There may be questions:

(1) Why such big difference between $|A_L|$ and $|A_T|$?

γ/Z-exchange terms are suppressed in both $\sigma(++)$ and $\sigma(--)$ since they are vectors (h and H are scalars of course!!).

$$\Downarrow$$

In A_L, not only the numerator $\sigma(++) - \sigma(--)$ but also the denominator receives little contribution from γ/Z exchange terms,

while in A_T only the numerator becomes small.

(2) Why A_T changes its sign while A_L not around $s = M_H^2$?

$\sigma(\chi = \pm\pi/2)$ receive sizable contributions from γ/Z exchange terms.

$$\Downarrow$$

The interference between the γ/Z-exchange and H-exchange terms, which is proportional to the H propagator, is important in A_T, and its sign changes thereby depending on whether $s > M_H^2$ or $s < M_H^2$.

On the other hand,
the sign of A_L is determined by the difference of $|\mathcal{M}_H|^2$.

Here we chose $a_\mu = b_\mu$ as an illustration, which makes $\sigma(++)$ positive, but different parameter choice, e.g., $a_\mu = -b_\mu$ could make A_L negative.

Detectability of the asymmetry

For $\sqrt{s} = m_H$ =500 GeV with $\mathrm{Re}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Im}\, b_{t,\mu}$ $= 0.2$,

$$A_L = 0.73,$$

while the cross sections are

$$\sigma(++) = 5.0 \times 10^{-2} \text{ fb and } \sigma(--) = 7.8 \times 10^{-3} \text{ fb},$$

leading to $N \simeq 29\epsilon$ events for an integrated luminosity $L = 500$ fb^{-1}, where we expressed the detection efficiency of $t\bar{t}$ productions as ϵ. They are combined to give the following statistical uncertainty:

$$\delta A_L = \sqrt{(1 - A_L^2)/N} = 0.68/\sqrt{\epsilon L} = 0.13/\sqrt{\epsilon}. \tag{15}$$

Consequently, the expected statistical significance N_{SD} is

$$N_{SD} \equiv |A_L|/\delta A_L = 5.7\sqrt{\epsilon}. \tag{16}$$

We can confirm $|A_L| \neq 0$ at $5.7\sqrt{\epsilon}$ level. $\Rightarrow$ $N_{SD} = 4.0$ for $\epsilon = 0.5$.

Assuming $L = 500$ fb^{-1} may be a bit too optimistic, but we used this value considering that we aim to find the possibility and limit of the muon colliders as mentioned in the beginning of this section. It is easy to transform our numerical results for any other L.

For example, for $L = 50$ fb^{-1},

$$N_{SD} = 1.8\sqrt{\epsilon} = 1.3 \text{ for } \epsilon = 0.5.$$

Parameter dependence of the asymmetry

$\sqrt{s}$ (GeV)	(a)			(b)			(c)		
	A_L	N	N_{SD}	A_L	N	N_{SD}	A_L	N	N_{SD}
450	0.08	7.6	0.2	0.40	11.7	1.5	0.73	25.7	5.4
480	0.19	9.4	0.6	0.64	21.1	3.8	0.88	61.8	14.4
500	0.26	10.5	0.9	0.73	28.7	5.7	0.92	91.6	21.8
520	0.23	10.1	0.7	0.69	25.2	4.8	0.90	77.8	18.2
550	0.12	8.8	0.4	0.51	15.9	2.4	0.81	40.9	8.9
600	0.05	7.7	0.1	0.29	10.4	1.0	0.63	19.8	3.6

Table 1: N_{SD} as a function of $\sqrt{s}$ (with $\epsilon = 1$ for simplicity) for $\mathrm{Re}\, a_{t,\mu} = \mathrm{Re}\, b_{t,\mu} = \mathrm{Im}\, a_{t,\mu} = \mathrm{Im}\, b_{t,\mu} = 0.1$ (a), 0.2 (b), and 0.3 (c)

$\sqrt{s}$ (GeV)	$\Gamma_H = \Gamma_h(m_H)$	$\Gamma_H = 80$ GeV	$\Gamma_H = 100$ GeV
450	1.5	1.4	1.3
500	5.7	4.4	3.2
550	2.4	2.3	2.1
600	1.0	1.0	1.0

Table 2: N_{SD} as a function of $\sqrt{s}$ for $\Gamma_H = 80$ and 100 GeV, where $\Gamma_H = \Gamma_h(m_H)$ means that Γ_H was computed with the SM formula ($\Gamma_h(m_H) = 67.5$ GeV).

	Re a_t	Re a_μ	Re b_t	Re b_μ	Im a_t	Im a_μ	Im b_t	Im b_μ
0.0	4.1	4.4	4.7	2.2	4.1	4.4	4.7	2.2
0.1	4.7	5.3	4.9	3.8	4.7	5.3	4.9	3.8
0.2	5.7	5.7	5.7	5.7	5.7	5.7	5.7	5.7
0.3	7.0	5.6	7.1	7.7	7.0	5.6	7.1	7.7

Table 3: N_{SD} as a function of each parameter for $\sqrt{s} = 500$ GeV with the rest being fixed to be 0.2

	$\mathrm{Re}\,\boldsymbol{a_t}$	$\mathrm{Re}\,\boldsymbol{a_\mu}$	$\mathrm{Re}\,\boldsymbol{b_t}$	$\mathrm{Re}\,\boldsymbol{b_\mu}$	$\mathrm{Im}\,\boldsymbol{a_t}$	$\mathrm{Im}\,\boldsymbol{a_\mu}$	$\mathrm{Im}\,\boldsymbol{b_t}$	$\mathrm{Im}\,\boldsymbol{b_\mu}$
0.0	1.1	2.0	2.0	0.4	2.3	1.5	1.9	1.8
0.1	1.7	2.3	2.1	1.3	2.3	1.9	2.0	2.1
0.2	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4
0.3	3.2	2.4	2.9	3.6	2.6	2.8	2.9	2.5

Table 4: N_{SD} as a function of each parameter for $\sqrt{s} = 550$ GeV with the rest being fixed to be 0.2

4. Optimal-observable analysis

The optimal-observable technique is a useful tool for estimating statistical uncertainties in various coupling measurements.

Suppose we have a cross section

$$\frac{d\sigma}{d\phi}(\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi), \qquad (17)$$

where $f_i(\phi)$ are known functions of the final-state variables ϕ and c_i's are model-dependent coefficients which are to be determined.

This can be done by using appropriate weighting functions $w_i(\phi)$ so that

$$\int d\phi \; w_i(\phi)\Sigma(\phi) = c_i.$$

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\Sigma(\phi) \tag{18}$$

This minimizes resultant statistical error, where X_{ij} is the inverse of

$$M_{ij} \equiv \int d\phi f_i(\phi) f_j(\phi)/\Sigma(\phi)\,. \tag{19}$$

With these weighting functions, the statistical uncertainty of c_i is

$$\delta c_i = \sqrt{X_{ii}\,\sigma_T/N}\,, \tag{20}$$

where $\sigma_T \equiv \int (d\sigma/d\phi) d\phi$ and N is the total number of events.

Here we focus on the longitudinal beam polarization.

We have altogether eight independent parameters:

$$\mathrm{Re}\, \boldsymbol{a}_{\mu,t}, \;\; \mathrm{Re}\, \boldsymbol{b}_{\mu,t}, \;\; \mathrm{Im}\, \boldsymbol{a}_{\mu,t}, \;\; \mathrm{Im}\, \boldsymbol{b}_{\mu,t}$$

It is too complicated to treat them all equally, so we assume

$$|\mathrm{Re}\, \boldsymbol{a}_{\mu,t}, \mathrm{Re}\, \boldsymbol{b}_{\mu,t}| \gg |\mathrm{Im}\, \boldsymbol{a}_{\mu,t}, \mathrm{Im}\, \boldsymbol{b}_{\mu,t}|$$

since the imaginary part of parameters (form factors) is often produced through higher-order-loop corrections in an underlying theory.

With this reduced parameter set and assumption, we have

$$\begin{aligned} &\frac{d}{d\cos\theta}\sigma_{++}(\mu\bar{\mu} \to t\bar{t}) \\ &\quad = f_{\mathrm{SM}}(\theta) + c_{aa} f_{aa}(\theta) + c_{ab} f_{ab}(\theta) + c_{ba} f_{ba}(\theta) + c_{bb} f_{bb}(\theta), \end{aligned} \tag{21}$$

where $f_{\mathrm{SM}}(\theta)$ expresses the SM contribution,

$$c_{aa} \equiv (\mathrm{Re}\, a_t)(\mathrm{Re}\, a_\mu), \quad c_{ab} \equiv (\mathrm{Re}\, a_t)(\mathrm{Re}\, b_\mu),$$
$$c_{ba} \equiv (\mathrm{Re}\, b_t)(\mathrm{Re}\, a_\mu), \quad c_{bb} \equiv (\mathrm{Re}\, b_t)(\mathrm{Re}\, b_\mu),$$

and $f_{ij}(\theta)$ $(i, j = a, b)$ are all independent of each other.

Practically, however, $f_{ia}(\theta)$ and $f_{ib}(\theta)$ are equivalent in the limit of $m_\mu \longrightarrow 0$ as

$$\begin{aligned} \bar{v}_+(p_{\bar{\mu}})(a_\mu + b_\mu\gamma_5)u_+(p_\mu) &\simeq \bar{v}_+(p_{\bar{\mu}})(a_\mu + b_\mu\gamma_5)\frac{1+\gamma_5}{2}u(p_\mu) \\ &= (a_\mu + b_\mu)\bar{v}_+(p_{\bar{\mu}})u_+(p_\mu) \end{aligned} \tag{22}$$

That is, a_μ and b_μ contribute almost equally to $d\sigma_{++}$, which leads to

$$f_{aa}(\theta) \simeq f_{ab}(\theta), \qquad f_{ba}(\theta) \simeq f_{bb}(\theta).$$

Therefore we neglect their differences and start from

$$\frac{d}{d\cos\theta}\sigma_{++}(\mu\bar{\mu} \to t\bar{t}) \simeq f_1(\theta) + c_a f_2(\theta) + c_b f_3(\theta), \tag{23}$$

where $c_a \equiv c_{aa} + c_{ab}$, $c_b \equiv c_{ba} + c_{bb}$, $f_1(\theta) = f_{\rm SM}(\theta)$, $f_2(\theta) = f_{aa}(\theta) \simeq f_{ba}(\theta)$ and $f_3(\theta) = f_{ba}(\theta) \simeq f_{bb}(\theta)$.

Using them for $\sqrt{s} = 550$ GeV and $m_H = 500$ GeV we obtained

$$\begin{aligned} &M_{11} = 7.75 \cdot 10^{-3},\ M_{12} = 5.80 \cdot 10^{-2},\ M_{13} = -2.38 \cdot 10^{-3}, \\ &M_{22} = 4.35 \cdot 10^{-1},\ M_{23} = -1.79 \cdot 10^{-2},\ M_{33} = 7.37 \cdot 10^{-4}, \end{aligned} \tag{24}$$

$\Rightarrow$ the $(2,2)$ and $(3,3)$ elements of X $(= M^{-1})$:

$$X_{22} = 3.68 \cdot 10^{6}, \quad X_{33} = 5.38 \cdot 10^{8}\,.$$

$\Rightarrow$ The expected statistical uncertainty in $c_{a,b}$ measurements are

$$\delta c_a = 1.92 \cdot 10^3/\sqrt{L}, \qquad \delta c_b = 2.32 \cdot 10^4/\sqrt{L}\,. \tag{25}$$

This means we need $L = 3.7 \cdot 10^6$ fb^{-1} for achieving $\delta c_a = 1$ and $L = 5.4 \cdot 10^8$ fb^{-1} for $\delta c_b = 1$, which are both far beyond our reach!

We then assume one of the parameters is determined in some other experiments.

(1) If c_a was unknown (i.e., if c_b was measured elsewhere),
the expected precision is $\delta c_a = 44.5/\sqrt{L}$, i.e., $\delta c_a = 1.99$ (6.29) for $L = 500$ (50) fb^{-1}.

(2) If c_b is undetermined (i.e., only c_a is known),
$\delta c_b = 539/\sqrt{L}$, i.e., $\delta c_b = 24.1$ (76.2) for $L = 500$ (50) fb^{-1}.

We also give results for some other $\sqrt{s}$ in Tables 5 and 6.

$\sqrt{s}$ (GeV)	δc_a	L =500/fb	L =50/fb
450	$41.1/\sqrt{L}$	1.84	5.81
480	$44.7/\sqrt{L}$	2.00	6.32
520	$44.7/\sqrt{L}$	2.00	6.32
580	$61.4/\sqrt{L}$	2.75	8.68

Table 5: Expected precision of c_a determination for $m_H = 500$ GeV

$\sqrt{s}$ (GeV)	δc_b	L =500/fb	L =50/fb
450	$227/\sqrt{L}$	10.2	32.1
480	$329/\sqrt{L}$	14.7	46.5
520	$447/\sqrt{L}$	20.0	63.2
580	$878/\sqrt{L}$	39.3	124.

Table 6: Expected precision of c_b determination for $m_H = 500$ GeV

Therefore, if $|c_a|$ is $O(1)$, there is some hope to catch new-physics signal thereby.
On the other hand, $|c_b|$ is required to be at least $O(10)$.

Note that it is not unrealistic to assume $|c_{a,b}|$ to be $O(1) \sim O(10)$ in various models with two (or multi) Higgs-doublets.

What we could know via A_L measurements is only on CP violation, while $c_{a,b}$ are both combinations of CP-conserving and CP-violating parameters.

Therefore they could work complementarily to each other.

5. Summary

We have carried out model-independent analyses of possible non-standard Higgs interactions with top/muon in $\mu\bar{\mu} \to t\bar{t}$.

Our main purpose here was to see if we could draw any useful information in the off-resonance region without depending on any specific models.

We computed two CP-violating asymmetries for longitudinal and transverse beam polarizations, and also studied whether we could determine the non-standard-coupling parameters separately through the optimal-observable (OO) procedure.

We found that A_L would be sizable, while A_T is too small to be a meaningful observable. We then estimated the detectability of A_L and showed that we would be able to observe some signal of CP violation as long as we are not too far from the H pole.

On the other hand, more detailed analyses via the OO procedure seem challenging. However, if we could reduce the number of unknown parameters with a help of other experiments, and if the size of the parameters is at least $O(1) \sim O(10)$, we might be able to get some meaningful information thereby.

***** Summary of Summary *****

Muon colliders could be useful for studying neutral Higgs sector through

(1) the longitudinal polarization asymmetry (for CP-violating coupling)

and

(2) the optimal-observable analyses (for CP-conserving plus CP-violating couplings)

Thank you!