Complete set of observables in 2HDM

I.F.Ginzburg, K.A.Kanishev, M.Krawczyk, D.Sokolovska

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Motivation

$$\mathcal{L}_\lambda(\phi) \quad \Rightarrow \quad \sigma_{ab\to cd}$$

- "Standard procedure".

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- The parameters λ can be detemined from observed data.

Motivation

$\mathcal{L}_\lambda(\phi)$

$\sigma_{ab \to cd}$

- "Symmetry breaking case".

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$$\mathcal{L}_\lambda(\phi) \quad \Rightarrow \quad \mathcal{L}_g(\langle\phi\rangle, \chi) \quad \Rightarrow \quad \sigma_{ab\to cd}$$

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- An "intermediate step" is introduced.

Motivation

$$\boxed{\mathcal{L}_{\lambda}(\phi)} \quad \Leftarrow \quad \boxed{\mathcal{L}_g(\langle\phi\rangle, \chi)} \quad \Leftarrow \quad \boxed{\sigma_{ab\to cd}}$$

- "Symmetry breaking case".
- An "intermediate step" is introduced.
- Need a procedure to "reverse" this arrow.

Motivation

$\mathcal{L}_{\lambda}(\phi)$ $\Leftarrow$ $\mathcal{L}_g(\langle\phi\rangle, \chi)$ $\Leftarrow$ $\sigma_{ab\to cd}$

In case of 2HDM

- Most general 2HDM contains 14 parameters (6 real and 4 complex)

? How one can find those parameters if masses and couplings are known?

? How many and which couplings are needed?

Outline

1. Two Higgs Doublet Model (2HDM)
   - 2HDM lagrangian
   - Extrema of the potential
   - Reparametrization and Higgs basis
2. Reconstruction: Quadratic terms
   - Mass matrix
   - Mixing matrix
3. Qubic and Quartic terms

$\Rightarrow$ Conclusions and outlook

2HDM Potential

- Two scalar doublets: $\phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}$
- Scalar combinations: $x_1 = \phi_1^\dagger \phi_1, \quad x_2 = \phi_2^\dagger \phi_2, \quad x_3 = \phi_1^\dagger \phi_2 \,.$

Most general form of potential

$$V = \tfrac{1}{2}\lambda_1 x_1^2 + \tfrac{1}{2}\lambda_2 x_2^2 + \lambda_3 x_1 x_3 + \lambda_4 x_3^\dagger x_3 + \left[\tfrac{1}{2}\lambda_5 x_3^2 + (\lambda_6 x_1 + \lambda_7 x_2)\, x_3 + h.c.\right]$$
$$-\tfrac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)\right]$$

Shorthand notations: $\lambda_{345} = \lambda_3 + \lambda_4 + Re\lambda_5$, $\tilde{\lambda}_{345} = \lambda_3 + \lambda_4 - Re\lambda_5$

- Most general case:λ_{1-4}, m_{11}^2, m_{22}^2 - real, λ_{5-7}, m_{12}^2 - complex.
- Particle content:
  - 3 neutral scalars h_1, h_2, h_3
  - one charged scalar $H^\pm$

Classification of extrema in 2HDM

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle\phi_2\rangle = \begin{pmatrix} u \\ v_2 e^{-i\xi} \end{pmatrix}, \quad u, v_1, v_2 > 0$$

$$v^2 = v_1^2 + v_2^2 + u^2, \quad \tan\beta = v_2/v_1$$

Neutral extremum: $u = 0$

Electric charge is conserved, 5 physical Higgs particles (h_1, h_2, h_3, and $H^{\pm}$)

Spontaneous CP violating (sCPv) $\xi \neq 0$ – two degenerate extrema

CP conserving (CPc) $\xi = 0$ – up to 4 such extrema

Electroweak conserving point (EW): $u = v_1 = v_2 = 0$

Electroweak symmetry is not broken. $M_{W,Z} = m_q = 0$

Charged extremum: $u \neq 0$

Electric charge is not conserved, photon is massive

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- v_1, v_2 and ξ are determined by system of cubic equations.
- "Shifted fields"

$$\phi_1 = \begin{pmatrix} G^\pm c_\beta - H^\pm s_\beta \\ \frac{v_1 + \eta_1 + i(G^0 c_\beta - \eta_3 s_\beta)}{\sqrt{2}} \end{pmatrix}, \phi_2 = \begin{pmatrix} (G^\pm c_\beta + H^\pm s_\beta) \\ \frac{v_2 + \eta_2 + i(G^0 s_\beta + \eta_3 c_\beta)}{\sqrt{2}} \end{pmatrix} e^{-i\xi}$$

- η_i – are not physical fields.

Reparametrization

- Model contains two fields with identical quantum numbers

$\Rightarrow$ It can be described in terms of fields ϕ_i', obtained from ϕ_i by a global unitary transformation.

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \mathcal{F} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\mathcal{F} = e^{-i\rho_0} \begin{pmatrix} \cos\theta e^{i\rho/2} & \sin\theta e^{i(\tau-\rho/2)} \\ -\sin\theta e^{-i(\tau-\rho/2)} & \cos\theta e^{-i\rho/2} \end{pmatrix}$$

- The transformation induces the changes of coefficients of Lagrangian – reparametrization transformation.

Higgs (Georgi) basis

- One can exploit reparametrization to set $v_2 = 0$: Higgs basis

- Notations for parameters in Higgs basis: $\lambda_i \to \Lambda_i$ and $m_{ij} \to \mu^2_{ij}$.
- Extremum equations:
$v^2\Lambda_1 = \mu^2_{11}$, $v^2 Re\Lambda_6 = Re\mu^2_{22}$, $v^2 Im\Lambda_6 = Im\mu^2_{22}$
- Decomposition:

$$\phi_1 = \begin{pmatrix} G^+ \\ \dfrac{v + \eta_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H^+ \\ \dfrac{\eta_2 + i\eta_3}{\sqrt{2}} \end{pmatrix}$$

η_i – are not physical states. Physical states: $h_i = R_{ij}\,\eta_j$.

Masses

Mass of the charged scalar

$$M^2_{H^\pm} = v^2\Lambda_3 - \mu^2_{22}$$

Neutral scalars mass matrix

$$M_{ij} = v^2 \begin{pmatrix} \Lambda_1 & Re\Lambda_6 & -Im\Lambda_6 \\ Re\Lambda_6 & \left[\Lambda_{345} - \mu^2_{22}/v^2\right]/2 & -Im\Lambda_5/2 \\ -Im\Lambda_6 & -Im\Lambda_5/2 & \left[\tilde{\Lambda}_{345} - \mu^2_{22}/v^2\right]/2 \end{pmatrix} =$$

$$= R^T \cdot \begin{pmatrix} M_1^2 & 0 & 0 \\ 0 & M_2^2 & 0 \\ 0 & 0 & M_3^2 \end{pmatrix} \cdot R$$

- Knowing $M_1, M_2, M_3,\ M_{H^\pm}$ and mixing matrix R one can find $\Lambda_1, \Lambda_4, Re\Lambda_5, Im\Lambda_5, Re\Lambda_6, Im\Lambda_6$

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$$
\begin{aligned}
v^2\Lambda_1 &= M_{11} && = R_{i1}R_{i1}M_i^2 \\
v^2\Lambda_4 &= M_{22} + M_{33} - M_{H^\pm}^2 && = (R_{i2}R_{i2} + R_{i3}R_{i3})\, M_i^2 + M_{H^\pm}^2 \\
v^2 Re\Lambda_5 &= M_{22} - M_{33} && = (R_{i2}R_{i2} - R_{i3}R_{i3})\, M_i^2 \\
v^2 Im\Lambda_5 &= -2M_{23} && = -2R_{i2}R_{i3}M_i^2 \\
v^2 Re\Lambda_6 &= M_{13} && = R_{i1}R_{i3}M_i^2 \\
v^2 Im\Lambda_6 &= -M_{23} && = -R_{i2}R_{i3}M_i^2
\end{aligned}
$$

(Summation over repeated indices implied.)

$$\mu_{11}^2 = v^2\Lambda_1, \qquad Re\mu_{22}^2 = v^2 Re\Lambda_6, \qquad Im\mu_{22}^2 = v^2 Im\Lambda_6$$

Rotation matrix

Rotation matrix is parametrized by 3 Euler angles.

Vector boson couplings

- $g(W^+W^-h_i) = -iR_{1i}\frac{g^2 v}{2}, \quad g(ZZh_i) = -iR_{1i}\frac{(g^2+(g')^2)v}{2}$
- $g(W^+H^-h_i) = -i(R_{2i} - iR_{3i})\frac{g}{2}(p_{H^+} - p_{h_i})$

Relative couplings

- Relative couplings $\chi_j^{(i)} = g_j^{(i)}/g_j^{"SM"}$
- $\chi_V^{(i)} = R_{1i}$
- $\chi_{H+W-}^{(i)} = R_{2i} - iR_{3i}$

Rotation matrix

- Elements of R_{ij} and M_{ij} are parametrization dependent.
- There is still reparametrization freedom in Higgs basis

to change the phase of the ϕ_2 field:

$$\phi_2 \to e^{i\rho}\phi_2$$

This corresponds to

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\rho & -\sin\rho \\ 0 & \sin\rho & \cos\rho \end{pmatrix} \cdot \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

- Only two Euler angles in R_{ij} are relevant.

Rotation matrix

- Example: $\chi_V^{(1)}$ and $\chi_V^{(2)}$ are known

$$R_{ij} = \begin{pmatrix} \chi_1 & \chi_2 & \sqrt{1-\chi_1^2-\chi_2^2} \\ -\chi_1\sqrt{\frac{1-\chi_1^2-\chi_2^2}{\chi_1^2+\chi_2^2}} & -\chi_2\sqrt{\frac{1-\chi_1^2-\chi_2^2}{\chi_1^2+\chi_2^2}} & \sqrt{\chi_1^2+\chi_2^2} \\ \frac{\chi_2}{\sqrt{\chi_1^2+\chi_2^2}} & -\frac{\chi_1}{\sqrt{\chi_1^2+\chi_2^2}} & 0 \end{pmatrix}$$

Triple couplings

$$M^2_{H^\pm} = v^2\Lambda_3 - \mu^2_{22}$$

$$M_{ij} = v^2 \begin{pmatrix} \Lambda_1 & Re\Lambda_6 & -Im\Lambda_6 \\ Re\Lambda_6 & \left[\Lambda_{345} - \mu^2_{22}/v^2\right]/2 & -Im\Lambda_5/2 \\ -Im\Lambda_6 & -Im\Lambda_5/2 & \left[\tilde{\Lambda}_{345} - \mu^2_{22}/v^2\right]/2 \end{pmatrix}$$

- Parameters $\Lambda_2, Re\Lambda_7, Im\Lambda_7$ don't appear in masses.

⇒ One could fix those parameters only from higgs self-couplings.

- Λ_3 and μ^2_{22} appear in masses only in combination $v^2\Lambda_3 - \mu^2_{22}$

⇒ One also need to obtain either Λ_3 or μ^2_{22} from self-couplings.

Triple couplings

$H^+H^-h_i$ couplings

$$g(H^+H^-h_i) = v(R_{1i}\Lambda_3 + R_{2i}Re\Lambda_7 - R_{3i}Im\Lambda_7)$$

$h_ih_ih_i$ couplings

$$g(h_ih_ih_i) = \frac{3}{v}\left[M_i^2(2 - R_{1i}^2)R_{1i} + v^2(1 - R_{1i}^2)(R_{1i}\frac{\mu_{22}^2}{v^2} + R_{2i}Re\Lambda_7 - R_{3i}Im\Lambda_7)\right]$$

Three triple couplings are required to find $Re\Lambda_7, Im\Lambda_7, \Lambda_3$ and μ_{22}^2

Triple couplings

General expression for triple couplings

$$g(h_i h_j h_k) = \frac{3M_a^2 R_{a\alpha}}{v}\left[2R_{a\beta}\mathcal{R}_{ijk}^{1\alpha\beta} - R_{a1}\mathcal{R}_{ijk}^{11\alpha}\right] + 3vT_a\left[\mathcal{R}_{ijk}^{a\alpha\alpha} - \mathcal{R}_{ijk}^{a11}\right]$$

- Where $T_i = \left(\mu_{22}^2/v^2, Re\Lambda_7, -Im\Lambda_7\right)$, and

$$\mathcal{R}_{ijk}^{abc} = \frac{1}{6}\left(R_{ia}R_{jb}R_{kc} + R_{ia}R_{jc}R_{kb} + R_{ib}R_{ja}R_{kc} + R_{ib}R_{jc}R_{ka} + R_{ic}R_{ja}R_{kb} + R_{ic}R_{jb}R_{ka}\right)$$

Necessity of quartic coupling

Λ_2 do not appear in triple couplings either

At least one quartic coupling is requred

$$g(H^+H^+H^-H^-) = 2\Lambda_2$$

$$g(h_i h_j h_k h_l) =$$
$$= \frac{3M_a^2}{v^2}\left[4R_{a\alpha}\left(R_{a\beta}\mathcal{R}_{ijkl}^{11\alpha\beta} - R_{a1}\,\mathcal{R}_{ijkl}^{111\alpha}\right) + R_{a1}R_{a1}\mathcal{R}_{ijkl}^{1111}\right] + B_{\alpha\beta}\left[\mathcal{R}_{ijkl}^{\alpha\beta\gamma\gamma} - \mathcal{R}_{ijkl}^{\alpha\beta 11}\right]$$

With $B_{1i} = B_{i1} = T_i,\ B_{22} = B_{33} = \Lambda_2/2.$

Constraints

Λ_i, μ_{ij}^2 **must also satisfy**

- Positivity constraints –potential is bounded from below.
- Vacuum stability –second minimum (if exists) is not lower.
- Unitarity.

On can also check the symmetries of potential.

Construction and diagonalization of $\Lambda_{\mu\nu}$ matrix [Ivanov],

is a simple numerical task.

CP conserving case

If potential had explicit CP consevation and there is no spontaneous CP violation
then in Higgs basis all parameters are real.

- $h_3 = \eta_3 = A$ – doesn't mix with h_1 and h_2

$$R_{ij} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- There's no CP violating couplings, like $g(AAA)$ or H^+H^-A.
  - (Though, generally, it is possible to have "CP violation in couplings")

$\Rightarrow$ Only one mixing angle and two (instead of three) triple couplings are required.

Conclusions

- In order to fix the parameters of most general 2HDM one needs
  - Rotation matrix (2 independent parameters).
  - Masses of 3 neutral and one charged scalars.
  - Three triple higgs self-couplings.
  - One quartic higgs self-coupling.
- In CP conserving case
  - Rotation matrix has only one independent parameter.
  - Only two triple higgs self-couplings are needed.
  - One quartic higgs self-coupling is still requred.

On $\tan\beta$

- General form of Yukawa sector in 2HDM $Q_L^i = \begin{pmatrix} \bar{u}_L^i \\ \bar{d}_L^i \end{pmatrix}$

$$-\mathcal{L}_Y = \Gamma_1^{ij} Q_L^i \phi_1 \, d_R^j + \Delta_1^{ij} Q_L^i \tilde{\phi}_1 \, u_R^j + \Gamma_2^{ij} Q_L^i \phi_2 \, d_R^j + \Delta_2^{ij} Q_L^i \tilde{\phi}_2 \, u_R^j + h.c.$$

- $\tan\beta$ is parametrization dependent.
  For example, in Higgs basis $\tan\beta = \dfrac{v_2}{v_1} = 0$

- There is a preferred reparametrization basis, where:

Model I

All fermions couple to ϕ_1
$\Delta_2 = \Gamma_2 = 0$

Model II

d_R couples to ϕ_1
u_R couples to ϕ_2
$\Delta_1 = \Gamma_2 = 0$

- "Usual" $\tan\beta$ is defined in these preferred basis