The Little Hierarchy Problem in a Generalized NMSSM

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by A. Delgado, CK, J.P. Olson and A. del la Puente
Plus a forthcoming work
Two major problems motivate much of SUSY model-building:

- **SUSY flavor (& CP) problem** = *How do we add scalar superpartners without generating large, new FCNC’s and CPV?*
  
  ⇒ *insert your favorite here* – mediated SUSY breaking

- **Little hierarchy problem (LHP)** = *How do we push light Higgs mass above the LEP bound (114 GeV) without heavy stops (∼ 1 TeV) and/or large $A_t$ ($\sim \sqrt{6}m_\tilde{t}$)?*

  Extend Higgs sector
  Extend symmetries of MSSM
  Impose strong couplings
  Impose low cutoff
  Add new operators
  Hide Higgs from LEP
The Little Hierarchy Problem

Weak-scale SUSY postulated to solve (big) hierarchy problem:

\[
\delta m_{\text{weak}}^2 = \delta H - H_t - H + H \sim H_t
\]

\[
= \frac{3y^2}{8\pi^2} (\Lambda^2 - \Lambda^2) + \frac{3y^2}{16\pi^2} m_t^2 \log \left( \frac{\Lambda}{m_{\text{weak}}} \right)
\]

To maintain \( m_{\text{weak}}^2 \), need \( m_t \lesssim 1 \text{ TeV} \).

BUT, the physical Higgs mass scales differently:

\[
m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{8\pi^2 v^2} \log \left( \frac{m_t^2}{m_t^2} \right)
\]

To get \( m_h \gtrsim 114 \text{ GeV} \) requires logarithmically large \( m_t \), but that destabilizes weak scale quadratically!
The NMSSM & Little Hierarchy Problem

Classic extension of MSSM \(\implies\) the Next-to-MSSM (NMSSM):

\[
W = W_{\text{Yukawa}} + \lambda SH_u H_d + \frac{1}{3} \kappa S^3
\]

Many advantages over MSSM:

- No \(\mu\)-term! Generated by \(\mu_{\text{eff}} = \lambda \langle S \rangle\).
- New quartic term in \(V\) from \(F_S\):
  \[
  |F_S|^2 = |\lambda H_u H_d + \kappa S^2|^2 = |\lambda|^2 |H_u H_d|^2 + \cdots
  \]
- New upper bound on \(m_{h^0}\):
  \[
  m_{h^0}^2 \leq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta
  \]

(Haber & Sher; Drees; Espinosa & Quiros; Kane, Kolda & Wells; \cdots)
Several problems & constraints as a solution to LHP:

**Low tan β Only**
Since $\Delta m_{h^0}^2 \propto \sin^2 2\beta$, important only when $\tan \beta \simeq 1$, where $m_{h^0} \to 0$ in MSSM.

**Perturbative Unification**
Assuming gauge coupling unification is real, want $\lambda$ perturbative up to GUT scale. But

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2} \left( 3y_t^2 + 4\lambda^2 + 2\kappa^2 - 3g_2^2 + \cdots \right)$$

Demanding $\lambda(M_{GUT}) \lesssim 4\pi$ requires $\lambda(m_W) \lesssim 0.7$.

**Higgs-Singlet Mixing**
Any mixing of singlet into $h^0$ decreases mass
- must tune mass matrix parameters to suppress mixing
- no one term controls mixing!
In large $m_{A^0}$ limit of NMSSM, CP-even scalar matrix takes form:

$$M^2 = \begin{pmatrix}
m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & (m_Z^2 - \lambda^2 v^2) \sin 2\beta \cos 2\beta \\
- & -
\end{pmatrix}
$$

where $M^2_{i3}$ are all naturally $O(M^2_{SUSY}) \approx O(m^2_W)$.

In particular,

$$M^2_{13} \propto 2\lambda v_s - (A_\lambda + 2\kappa v_s) \sin 2\beta$$

Any $S - h^0$ mixing will reduce $m_{h^0}$, so we need $M^2_{13} \simeq 0$:

$$\longrightarrow A_\lambda \simeq \left( \frac{2\lambda}{\sin 2\beta} - 2\kappa \right) v_s.$$
A typical NMSSM case: \((\lambda = 0.7, \kappa = 0.05, M_{\tilde{g}} = 500 \text{ GeV}, m_{\tilde{t}} = 1 \text{ TeV}, A_t = \sqrt{6}m_{\tilde{t}})\)

\[ \Rightarrow A_\lambda \text{ must be tuned to get EW symmetry breaking, and even more to get } m_{h^0} \text{ above LEP bound.} \]
The problem?
Maybe we are asking too much of the singlet

▶ Solve the $\mu$-problem
▶ Solve the little hierarchy problem

The S-MSSM
Allow (almost) all possible terms in $W$:

$$W = W_{\text{Yukawa}} + (\mu + \lambda S)H_uH_d + \frac{1}{2}\mu_s S^2 + \frac{1}{3}\kappa S^3$$

▶ Assume $\mu \sim m_W$. $S$ cleanly decouples to MSSM as $\mu_s \to \infty$. We assume $\mu_s$ large compared to other weak-scale masses.
▶ For simplicity, take $\kappa \sim 0$ – wouldn’t usually play big role anyway.

Not the final UV theory, but may describe low-E effective theory.
Why isn’t this model already well-studied?

- It doesn’t solve $\mu$-problem! Instead it has two $\mu$-problems!
  ⇒ We study lots of models with $\mu$-problems, like CMSSM, GMSB, AMSB, ... 
- We’ve left out dangerous tadpole terms which can’t be eliminated by symmetries, and these destabilize the hierarchy.

In usual NMSSM, $W$ possesses $Z_3$ symmetry, prevents tadpole terms:

$$W \sim \xi S, \quad V_{soft} \sim \xi' S$$

But in S-MSSM, $Z_3$ broken (softly) by $\mu, \mu_s$. At $n$-loops, one generates tadpoles with

$$\xi \sim \left( \frac{1}{16\pi^2} \right)^n \mu(s) \Lambda, \quad \xi' \sim \left( \frac{1}{16\pi^2} \right)^n M_{SUSY} \mu(s) \Lambda$$

If $\Lambda \gg m_W$, hierarchy can be destabilized!
The bad and not-so-bad about the tadpoles:

- Setting $\xi = 0$ in $W$ is technically natural. We only generate it by integrating out heavy fields, $W = S\Phi \Phi$. This can also be disallowed in a technically natural way.

- SUSY-breaking $\xi'$-tadpole is more dangerous. For example, can arise from non-minimal Kähler potential:

\[ K = S^\dagger S + (S + S^\dagger)\Phi^\dagger \Phi / M_{PL} \longrightarrow \xi' \sim F_\Phi^2 / M_{PL} \]

... or from coupling to heavy fields induced by supergravity.

- BUT $S$ need not be singlet all the way to $M_{PL}$. $\Lambda$ could be low.
- Adding new field(s) whose vev give $\mu, \mu_S$ would restore $Z_3$.
- Discrete $R$-symmetries can rule out tadpole contributions.

- We assume that S-MSSM is valid below some scale $\Lambda$ which may not be much greater than $m_W$ OR there is a discrete $R$-symmetry present.
The Potential of the S-MSSM

\[ V = (m_{H_u}^2 + |\mu + \lambda S|^2)|H_u|^2 + (m_{H_d}^2 + |\mu + \lambda S|^2)|H_d|^2 + (m_S^2 + \mu_S^2)|S|^2 \\
+ \left[ B_S S^2 + (\lambda \mu_S S^\dagger + B_\mu + \lambda A S) H_u H_d + h.c. \right] + \lambda^2 |H_u H_d|^2 \\
+ \frac{1}{8} (g^2 + g'^2) \left( |H_u|^2 - |H_d|^2 \right)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2. \]

- Three soft scalar masses, two \( B \)-terms, one \( A \)-term
- New quartic coupling will raise Higgs mass!
The Potential of the S-MSSM

\[ V = (m_{H_u}^2 + |\mu + \lambda S|^2)|H_u|^2 + (m_{H_d}^2 + |\mu + \lambda S|^2)|H_d|^2 + (m_s^2 + \mu_s^2)|S|^2 \\
+ \left[ B_s S^2 + (\lambda \mu_s S^\dagger + B_\mu + \lambda A_\lambda S) H_u H_d + h.c. \right] + \lambda^2 |H_u H_d|^2 \\
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+ \left[ B_s S^2 + (\lambda \mu_s S^\dagger + B_\mu + \lambda A \lambda S) \right] H_u H_d + \text{h.c.} \right] + \lambda^2 |H_u H_d|^2 \\
+ \frac{1}{8}(g^2 + g'^2) \left( |H_u|^2 - |H_d|^2 \right)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2. \]

Minimization conditions:

1. \[ \frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu_{\text{eff}}^2, \]

2. \[ \sin 2\beta = \frac{2 B_{\mu,\text{eff}}}{m_{H_u}^2 + m_{H_d}^2 + 2 \mu_{\text{eff}}^2 + \lambda^2 v^2} \]

where \( v_{s,u,d} = \langle \{S, H_u, H_d\} \rangle \), with \( v = (v_u^2 + v_d^2)^{1/2} = 174 \text{ GeV} \).

\[ \mu_{\text{eff}} = \mu + \lambda v_s, \]

\[ B_{\mu,\text{eff}} = B_\mu + \lambda v_s (\mu_s + A_\lambda). \]
The Potential of the S-MSSM

\[ V = (m_{H_u}^2 + |\mu + \lambda S|^2)|H_u|^2 + (m_{H_d}^2 + |\mu + \lambda S|^2)|H_d|^2 + (m_s^2 + \mu_s^2)|S|^2 \\
+ [B_sS^2 + (\lambda \mu_s S^\dagger + B_\mu + \lambda A_\lambda S) H_u H_d + h.c.] + \lambda^2 |H_u H_d|^2 \\
+ \frac{1}{8}(g^2 + g'^2) \left(|H_u|^2 - |H_d|^2\right)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2. \]

Minimization conditions:

\[ v_s = \frac{\lambda v^2}{2} \frac{(\mu_s + A_\lambda) \sin 2\beta - 2\mu}{\mu_s^2 + \lambda^2 v^2 + m_s^2 + 2B_s} \]

\[ \approx \frac{\lambda v^2}{2\mu_s} \sin 2\beta \quad \text{for large } \mu_s \]

\[ \longrightarrow 0 \quad \text{as } \mu_s \rightarrow \infty \]

Unlike NMSSM:

\[ v_s \text{ typically quite small.} \]

\[ \text{breaks EW symmetry very generically – conditions same as in MSSM, no additional tunings required.} \]
Scalar Masses

CP-even mass matrix similar to NMSSM. In particular:

\[ M_{11}^2 = m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin 2\beta \]

But:

\[ M_{13}^2 \approx -\lambda v \mu_s \sin 2\beta + \cdots \]
\[ M_{33}^2 \approx \mu_s^2 \]

Both good and bad:

- **S – h^0 mixing → 0 as \( \mu_s \to \infty \)**
- **All effects of S on mass matrix decouple as \( \mu_s \to \infty \)**!
- **We want to live in intermediate regime – is this fine tuned?**
Find Higgs spectrum as an expansion in $1/\mu_s$:

$$
m^2_{A_1^0} \cong \frac{2B_\mu}{\sin 2\beta} + \frac{2\lambda^2 v^2}{\mu_s} \left(2A_\lambda - \frac{\mu}{\sin 2\beta}\right)
$$

$$
m^2_{A_2^0, H_2^0} \cong \mu_s^2 + 2\lambda^2 v^2 + m_s^2 \mp 2B_s
$$

$$
m^2_{h_0^0, H_1^0} \cong m^2_{h_0^0, H_1^0}^{\text{MSSM}} + \frac{2\lambda^2 v^2}{\mu_s} (\mu \sin 2\beta - A_\lambda \mp \Delta)
$$

where

$$
\Delta = \frac{A_\lambda (m_Z^2 - m_{A_1^0}^2) \cos^2 2\beta - \mu (m_{A_1^0}^2 + m_Z^2) \sin 2\beta}{\sqrt{(m_{A_1^0}^2 + m_Z^2)^2 - 4m_{A_1^0}^2 m_Z^2 \cos^2 2\beta}}
$$

In Higgs decoupling limit, $m_{A_1^0} \to \infty$, mass of $h^0$ maximized:

$$
m_{h_0}^2 \cong m_Z^2 \cos^2 2\beta + \frac{2\lambda^2 v^2}{\mu_s} \left(2\mu \sin 2\beta - A_\lambda \sin^2 2\beta\right) + \cdots
$$
In principle, $m_{h^0} \to \infty$ as $\lambda \to \infty$.

BUT if we apply perturbative unification constraint, maximum $\lambda$ depends on $\tan \beta$:

Max $\lambda$ falls off quickly for $\tan \beta \lesssim 2$ or $\gtrsim 50$. 
For $m_t = M_{\tilde{g}} = 2\mu = 1$ TeV, $A_t = \sqrt{6}m_t$ (max mixing), $A_\lambda = \pm 1$ TeV and $\mu_s = 2$ TeV:

- All masses calculated using full one-loop $V_{\text{eff}}$ plus leading 2-loop corrections from FeynHiggs.
- Because of $\sin 2\beta$ term, effect persists to higher $\tan \beta$ than NMSSM.
- Different signs of $A_\lambda$ dominate at different $\tan \beta$ due to $1/\mu_s^2$ terms.
- Enhancement disappears as $\tan \beta \rightarrow 1$ due to perturbative unification constraint on $\lambda$, and MSSM contribution going to zero.
Can we bring down the stop masses?

For maximal mixing scenario: \((\mu = 500 \text{ GeV}, \mu_s = 2 \text{ TeV})\)

- ★ Even for \(m_t \approx 400 \text{ GeV}, \) S-MSSM produces \(h^0\) well above LEP bound.
Dependence on $\mu_s$: ($m_{t_2} = 1$ TeV, $A_t \sim 0$)

- Falls quickly as $\mu_s \to m_W$, falls slowly as $\mu_s \to \infty$.
- For maximum $m_{h^0}$, S-MSSM prefers $\mu_s$ 2 to 4 times larger than $\mu$.
- But choice of $\mu_s$ is not very tuned – wide ranges work!
What’s been accomplished so far:

**Broken EW symmetry naturally**

Assuming $\mu_s$ not very small, $V(S)$ stabilized by $\mu_s$ term, $\langle S \rangle$ small. No cancellations among parameters required. Vacuum structure is very MSSM-like.

**NOT solved $\mu$-problem**

Gave mass to charginos/neutralinos with explicit $\mu$-term.

**Raised the light Higgs mass**

For large, but not too large, values of $\mu_s$, we have raised $m_{h^0}$ to as much as 140 GeV, with no tunings among parameters required.

**Hard to tell from MSSM**

Phenomenology is essentially that of MSSM, except $m_{h^0}$ is too big given “observed” stop masses.

**But . . .**

Will this survive embeddings into a more complete model, e.g., a SUSY-breaking scheme?
Gauge-Mediated S-MSSM

To test S-MSSM in more complete theory, embed into gauge-mediated scheme:

\[ W = W_{SMSSM} + X \Phi \Phi \]

with \( \langle X \rangle = M + \theta^2 F \) and messengers \( \Phi \), \( \Phi \) in \( 5, \bar{5} \) of SU(5).

For S-MSSM soft masses:

\[
M_i(M) = \frac{\alpha_i}{4\pi} \frac{F}{M}
\]

\[
m^2_f(M) = \sum_{i=gauge} 2C_i \frac{\alpha_i^2}{16\pi^2} \left( \frac{F}{M} \right)^2
\]

\[ A_{\lambda, Q, ...} \approx 0 \]

\[ B_s, m^2_s \approx 0 \]

We obtain \( B_\mu, \mu \) from EWSB conditions

\[ \Rightarrow \text{We do NOT solve } \mu - B_\mu \text{ problem of GMSB.} \]
Random scan of parameter space:

For $M_{SUSY} = 500$ GeV, half of points above LEP bound.

GMSB models usually require $M_{SUSY} > 2$ TeV because $A_t \simeq 0$.

$$2 \leq \tan \beta \leq 6$$
$$2 \leq \mu_s/\mu \leq 5$$
$$300 \text{ GeV} \leq \mu \leq 900 \text{ GeV}$$

$$M = 10^{10} \text{ and } 10^{13} \text{ GeV}$$

$$M_{SUSY} = \sqrt{m_{t_1} m_{t_2}}$$
Recap of argument so far:

- We want contributions to $m_h$ from $F_S$ (raises $m_h$), but we don’t want $S$ mixing into $h$ (lowers $m_h$).
- Mixing is controlled by $\mu_S$. As $\mu_S$ grows, mixing decreases, but so do $F_S$ contributions to $m_h$.
- For $\mu_S$ of 1 to 5 TeV, mixing is small but $F_S$ contributions still sizable, able to push $m_h$ over 130 to 140 GeV.
- Little hierarchy problem is solved!

- But what if $\mu_S \ll m_W$?? It would appear to be a disaster, with large $h - S$ mixing, and two light states for LEP to find.
  ⇒ That’s not the case!
Consider minimization of potential again:

\[
\lambda v_s = \frac{\lambda^2 v^2}{2} \frac{(\mu_s + A_\lambda) \sin 2\beta - 2\mu}{\lambda^2 v^2 + \mu^2_s + m^2_s + 2B_s}
\]

\[
\approx \frac{1}{2} A_\lambda \sin 2\beta - \mu \quad \text{for small } \mu^2_s, m^2_s, B_s
\]

\[
\Rightarrow \mu_{\text{eff}} \approx \frac{1}{2} A_\lambda \sin 2\beta \text{ independent of } \mu!
\]

The heavy, MSSM-like pseudoscalar has mass

\[
m^2_A \approx \frac{2B_{\mu,\text{eff}}}{\sin 2\beta} + \lambda^2 v^2 \quad (A_\lambda \gg B_\mu)
\]

where

\[
B_{\mu,\text{eff}} \approx B_\mu + \frac{1}{2} A^2_\lambda \sin 2\beta - \mu A_\lambda
\]

Note that we can arrange cancellations among \(B_\mu, A_\lambda\) and \(\lambda v\) to obtain very light \(A^0\) (as in Dermisek & Gunion), but doesn’t come out automatically.
For scalar mass matrix, rotate by angle $\beta$ and work in "Goldstone" / Higgs decoupling basis:

$$
M^2 = \begin{pmatrix}
  m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & (m_Z^2 - \lambda^2 v^2) \sin 2\beta \cos 2\beta & 0 \\
  m_A^2 + (m_Z^2 - \lambda^2 v^2) \sin^2 2\beta & \lambda v A_\lambda \cos 2\beta & \lambda^2 v^2
\end{pmatrix}
$$

For small singlet mass terms $(m_s^2, B_s, \mu_s)$ and large $m_A$, the singlet does not mix into the SM-like Higgs at all!

The $F_S$ contributions to $m_h$ are nearly maximized. Only suppression is usual mixing of $H^0$ into $h^0$, pushing down the $h^0$ mass:

$$
m_h^2 = m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{(m_Z^2 - \lambda^2 v^2)^2}{m_A^2} \sin^2 2\beta \cos^2 2\beta
$$

And even this mixing is smaller than usual: $m_Z^2$ vs. $m_Z^2 - \lambda^2 v^2$. 
For scalar mass matrix, rotate by angle $\beta$ and work in "Goldstone" / Higgs decoupling basis:

$$
M^2 = \begin{pmatrix}
  m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & (m_Z^2 - \lambda^2 v^2) \sin 2\beta \cos 2\beta & 0 \\
  0 & m_A^2 + (m_Z^2 - \lambda^2 v^2) \sin^2 2\beta & \lambda v A \lambda \cos 2\beta \\
  m^2 & \lambda v A \lambda \cos 2\beta & \lambda^2 v^2
\end{pmatrix}
$$

For small singlet mass terms ($m_s^2, B_s, \mu_s$) and large $m_A$, the singlet does not mix into the SM-like Higgs at all!

The zero in $M^2_{13}$ is corrected by terms $\sim (m_s^2, B_s, \mu_s^2)/(\lambda^2 v^2)$. We seem to need small $m_s^2$, etc, while keeping $A_\lambda$ large. Not entirely natural because

$$
\frac{dm_s^2}{d \log Q} \sim \frac{1}{8\pi^2} \lambda^2 A_\lambda^2
$$

Suggests a model with low messenger scales (i.e., gauge mediation).
For scalar mass matrix, rotate by angle $\beta$ and work in “Goldstone" / Higgs decoupling basis:

$$
\mathcal{M}^2 = \begin{pmatrix}
 m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & (m_Z^2 - \lambda^2 v^2) \sin 2\beta \cos 2\beta & 0 \\
 0 & m_A^2 + (m_Z^2 - \lambda^2 v^2) \sin^2 2\beta & \lambda \nu A \lambda \cos 2\beta \\
 m_Z^2 \sin^2 2\beta & \lambda \nu A \lambda \sin 2\beta & \lambda^2 v^2
\end{pmatrix}
$$

For small singlet mass terms ($m_s^2, B_s, \mu_s$) and large $m_A$, the singlet does not mix into the SM-like Higgs at all!

BUT, we don’t really need to suppress $m_s^2, \mu_s, B_s$ too much. For example, turn on $m_s^2$:

$$
\delta m_h^2 \simeq \left( \frac{m_s^2}{m_A^2} \right) 2A\lambda \sin 2\beta (A\lambda \sin 2\beta - 2\mu).
$$

SO $m_h^2$ goes up or down depending on details. This is because $h$ is no longer lightest scalar eigenvalue!
For the mostly singlet particles:

\[ m^2_{A_s} \simeq \mu^2_s + \lambda^2 v^2 - \frac{\lambda^2 v^2 A^2}{m^2_A}, \]
\[ m^2_{h_s} \simeq \mu^2_s + \lambda^2 v^2 - \frac{\lambda^2 v^2 A^2}{m^2_A} \cos^2 2\beta \quad (m_{A_s} < m_{h_s}) \]

It would be quite natural for the associated singlinos to be lightest sparticles and thus dark matter candidates, but not studied in detail.
A random scatter of points with $B_\mu < 1000^2 \text{GeV}^2$, $A_\lambda < 700 \text{GeV}$, $\mu < 500 \text{GeV}$, $\mu_s < 50 \text{GeV}$, $m_s^2 = B_s = 0$

And: $\tan \beta = 2$, $m_t = 500 \text{GeV}$ and $A_t = 0 \rightarrow$ little stop mixing
A random scatter of points with $B_\mu < 1000^2 \text{GeV}^2$, $A_\lambda < 700 \text{GeV}$, $\mu < 500 \text{GeV}$, $\mu_s < 50 \text{GeV}$, $m_s^2 = B_s = 0$

And: $\tan \beta = 2$, $m_t = 500 \text{GeV}$ and $A_t = 0 \rightarrow$ little stop mixing
We can also plot some inputs with varying stop masses and $A_t$ between no-mixing and max-mixing scenarios ($\tan \beta = 2$):

Key: Black = 114 GeV, Red = MSSM, Blue=S-MSSM
Finally we can vary $\tan \beta$ while sampling parameter space with $A_t = 0$:

Thus this model requires $\tan \beta \lesssim 4.5 - 5$ in order to push $m_h$ above LEP bound.
Can models such as these be motivated?

- From a $Z_{4R}$ or $Z_{8R}$ symmetry

G. Ross et al. have shown only two anomaly-free discrete symmetries forbid $B, L$-violating terms in $W$:

\[
\begin{array}{cccccc}
 & 10 & \bar{5} & H_u & H_d & S \\
Z_{4R} & 1 & 1 & 0 & 0 & 2 \\
Z_{8R} & 1 & 5 & 0 & 4 & 6 \\
\end{array}
\]

Under both groups, most general $W$ is $W_{\text{SMSSM}} + \kappa S^3$.

Under $Z_{8R}$:

\[
\frac{\mu_S}{\mu} = \frac{\kappa}{2\lambda}
\]

generating the light S-MSSM if $\kappa$ is small.
Can models such as these be motivated?

- As a Froggat-Nielsen model

Model has a softly-broken PQ symmetry when $\kappa \to 0$:

$$PQ(\lambda) = 0; \quad PQ(\mu) = -2; \quad PQ(\mu_s) = 4; \quad PQ(\kappa) = 6$$

Likewise for $A$, $B$-terms.

Suppose PQ symmetry was an exact symmetry broken by a vev $\Theta$ and communicated to MSSM at a scale $M$ where $\Theta/M \sim O(1/10)$. Then we expect:

$$\lambda \sim 1 \gg \kappa \quad \text{and} \quad A_\lambda > \mu, B_\mu > \mu_s, B_s > A_\kappa$$

This, plus $m_s^2 \ll \lambda^2 v^2$, is what we need for the light S-MSSM case.
Conclusions

\[ \text{S-MSSM} = \begin{cases} 
\text{Generalized NMSSM, with} \\
\text{explicit supersymmetric mass terms} \\
\text{at or near weak scale}
\end{cases} \]

By sacrificing the solution to \( \mu \)-problem, the S-MSSM:

- Eliminates tunings among parameters in NMSSM to break EW symmetry and raise Higgs mass and solve little hierarchy problem.
- Pushes the Higgs mass above LEP bound (up to 140 GeV) for wide ranges of \( \mu_s \gtrsim 1 \text{ TeV}, \tan \beta \lesssim 10 \) and \( m_\tilde{t} \gtrsim 300 \text{ GeV} \).
- At LHC, singlet will not be seen, but effects will be seen through Higgs mass which is too heavy given observed SUSY spectrum.
- Embeds easily into gauge-mediated SUSY-breaking scheme, producing Higgs masses over 120 GeV for fairly generic parameters and \( m_\tilde{t} \) as low as 350 GeV.
- For \( \mu_s \ll \lambda v \), mostly-singlet scalars become lightest state but unseen at LEP. Doublet-singlet mixing is generically small, and can even raise SM-like Higgs mass. Typically pushes \( m_h \) up to 125 GeV. New light states present in model.