Observable Scalars from Neutrino Mixing

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Contents

- What is needed to explain Neutrino Mixing
- Frobenius Group T_7
- $U(1)_{B-L}$ Gauge Extension with T_7
- Lepton Triality and Higgs Structure
- Observable Scalars at the LHC
- Conclusion

What is needed to explain Neutrino Mixing

The usual way that neutrino mixing is explained is to say that in the basis where the charged leptons are diagonal, the neutrino mass matrix is of a certain form, perhaps with some texture zeros or with some symmetry such as $\mu - \tau$ reflection. However, this approach ignores the first indispensable step, i.e. how does the charged-lepton mass matrix gets diagonalized? This is important, because $(\nu, l)_L$ is a gauge doublet. Any symmetry defined on ν automatically applies to l. How can a symmetry be compatible with totally different m_e, m_μ, and m_τ?

This seemingly impossible problem is solved not by a symmetry alone, but by a symmetry (e.g. A_4) breaking into a residual symmetry (Z_3). The first such example was Ma/Rajasekaran(2001): Let $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$ under A_4 then the charged-lepton mass matrix is given by

$$M_l = \begin{pmatrix} y_1 v_1 & y_2 v_1 & y_3 v_1 \\ y_1 v_2 & \omega^2 y_2 v_2 & \omega y_3 v_2 \\ y_1 v_3 & \omega y_2 v_3 & \omega^2 y_3 v_3 \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$.

If $v_1 = v_2 = v_3 = v/\sqrt{3}$ is assumed, i.e. a residual Z_3 symmetry, then

$$M_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} v.$$

This means that M_l is diagonalized by a known matrix (determined by the group multiplication rules of A_4) regardless of m_e, m_μ, m_τ. After this special matrix was discovered in 2001, it was realized that it has already appeared long ago in the context of neutrino mixing!

In 1978, soon after the putative discovery of the third family of leptons and quarks, it was conjectured by Cabibbo and Wolfenstein independently that this special matrix could in fact be the neutrino mixing matrix. In hindsight, such would be the case if the neutrino mass matrix is diagonal in the original A_4 basis.
In the conventional parametrization, this means $\theta_{12} = \theta_{23} = \pi/4$, $\tan^2\theta_{13} = 1/2$, $\delta_{CP} = \pm\pi/2$, i.e. not everyone expected small lepton mixing angles.

In 2002, Harrison/Perkins/Scott proposed the tribimaximal hypothesis, i.e.

$$U_{l\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0)$$

This means $\sin^2 2\theta_{23} = 1$, $\tan^2\theta_{12} = 1/2$, $\theta_{13} = 0$.
In 2004, I showed that this tribimaximal mixing may be obtained in A_4, with

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix},$$

in the basis that M_l is diagonal.

At that time SNO data gave $\tan^2\theta_{12} = 0.40 \pm 0.05$, but it was changed in early 2005 to 0.45 ± 0.05. Tribimaximal mixing and A_4 then became part of the lexicon of the neutrino theorist.

At present, $\tan^2\theta_{12} = 0.47 \pm 0.05$, and a recent T2K measurement has indicated a nonzero θ_{13}.

Regardless of what the neutrino mass matrix turns out to be, the symmetry of the Yukawa sector for charged leptons remains Z_3, which may come from $A_4, T_7, \Delta(27)$ and many other possible discrete symmetries.

The challenge is to prove experimentally that this lepton triality exists. If A_4 or some other such symmetry is realized by a renormalizable theory at the electroweak scale, then the extra Higgs doublets required will bear this information. Specifically, A_4 breaks to the residual symmetry Z_3 in the charged-lepton sector, and all Higgs Yukawa interactions are determined in terms of lepton masses, as already shown by Ma/Rajasekaran(2001). This notion of lepton triality [Ma(2010)] is the key to such a proof, but how are these exotic Higgs doublets produced and detected?

Frobenius Group T_7

The tetrahedral group A_4 (12 elements) is the smallest group with a real $\underline{3}$ representation. The Frobenius group T_7 (21 elements) is the smallest group with a pair of complex $\underline{3}$ and $\underline{\overline{3}}$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$.

T_7 has been considered by Luhn/Nasri/Ramond(2007), Hagedorn/Schmidt/Smirnov(2009), and King/Luhn(2009).
The character table of T_7 (with $\xi = -1/2 + i\sqrt{7}/2$) is given by

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	$\chi_{\bar{3}}$
C_1	1	1	1	1	1	3	3
C_2	7	3	1	ω	ω^2	0	0
C_3	7	3	1	ω^2	ω	0	0
C_4	3	7	1	1	1	ξ	ξ^*
C_5	3	7	1	1	1	ξ^*	ξ

Multiplication rules:

$$\underline{3} \times \underline{3} = \underline{\bar{3}}(23, 31, 12) + \underline{\bar{3}}(32, 13, 21) + \underline{3}(33, 11, 22).$$

$$\underline{3} \times \underline{\bar{3}} = \underline{1}(1\bar{1} + 2\bar{2} + 3\bar{3}) + \underline{1}'(1\bar{1} + \omega 2\bar{2} + \omega^2 3\bar{3})$$
$$+\underline{1}''(1\bar{1} + \omega^2 2\bar{2} + \omega 3\bar{3}) + \underline{3}(2\bar{1}, 3\bar{2}, 1\bar{3}) + \underline{\bar{3}}(1\bar{2}, 2\bar{3}, 3\bar{1}).$$

Note that $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants,
and $\underline{3} \times \underline{3} \times \underline{\bar{3}}$ has one invariant.
These serve to distinguish T_7 from A_4 and $\Delta(27)$.

$U(1)_{B-L}$ Gauge Extension with T_7

Cao/Khalil/Ma/Okada(PRL 106, 131801 (2011)): Under T_7, let $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, $\tilde{\Phi} = (\bar{\phi}^0, -\phi^-)_i \sim \underline{\bar{3}}$. The Yukawa couplings $L_i l_j^c \tilde{\Phi}_k$ generate the charged-lepton mass matrix $M_l =$

$$\begin{pmatrix} f_1 v_1 & f_2 v_1 & f_3 v_1 \\ f_1 v_2 & \omega^2 f_2 v_2 & \omega f_3 v_2 \\ f_1 v_3 & \omega f_2 v_3 & \omega^2 f_3 v_3 \end{pmatrix} = U_{CW}^\dagger \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} v,$$

if $v_1 = v_2 = v_3 = v/\sqrt{3}$, as in the original A_4 proposal.

Let $\nu^c_i \sim \underline{\bar{3}}$, then the Yukawa couplings $L_i \nu^c_j \Phi_k$ are allowed, with

$$M_D = f_D v \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Note that Φ and $\tilde{\Phi}$ have $B - L = 0$.

Now add the neutral Higgs singlets $\chi_i \sim \underline{3}$ and $\eta_i \sim \underline{\bar{3}}$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu^c_i \nu^c_j \chi_k$ and $\nu^c_i \nu^c_j \eta_k$. Note that $\chi^*_i \sim \underline{\bar{3}}$ is not the same as $\eta_i \sim \underline{\bar{3}}$ because they have different $B - L$.

This means that both $B-L$ and the complexity of T_7 are required for this scenario. The heavy Majorana mass matrix for ν^c is then $M =$

$$h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u_3' & u_2' \\ u_3' & 0 & u_1' \\ u_2' & u_1' & 0 \end{pmatrix} = \begin{pmatrix} A & C & B \\ C & D & C \\ B & C & D \end{pmatrix},$$

where $A = hu_2$, $D = hu_1 = hu_3$ and $B = h'u_2'$, $C = u_1' = u_3'$ have been assumed, i.e. $T_7 \to Z_2$.
If $D = A$ and $C = 0$, then $\langle \chi_i \rangle \sim (1,1,1) \sim \langle \Phi \rangle$ and $\langle \eta_i \rangle \sim (0,1,0)$, which is the form discussed in Cao/Khalil/Ma/Okada(PRL 106, 131801 (2011)).

The seesaw neutrino mass matrix is now $M_\nu =$

$$-M_D M^{-1} M_D^T = \frac{-f_D^2 v^2}{A^3 - AB^2} \begin{pmatrix} A^2 - B^2 & 0 & 0 \\ 0 & A^2 & -AB \\ 0 & -AB & A^2 \end{pmatrix},$$

i.e. the two-parameter tribimaximal form proposed by Babu/He(2005), but without the auxiliary $Z_4 \times Z_3$ symmetry assumed there.

Cao/Khalil/Ma/Okada(arXiv:1108.0570): If $D \neq A$ and $C \neq 0$, deviations from tribimaximal mixing occur, with the prediction $\sin^2 2\theta_{23} \simeq 1 - \sin^2 2\theta_{13}/2$.

Lepton Triality and Higgs Structure

In the charged-lepton Yukawa sector, i.e. $L_i l^c_j \tilde{\Phi}_k$, a residual Z_3 symmetry exists so that $\Phi_k \to \phi_0, \phi_1, \phi_2 \sim 1, \omega, \omega^2$ ($\langle \phi^0_0 \rangle = v$) together with $e, \mu, \tau \sim 1, \omega^2, \omega$. Their interactions are given by

$$v^{-1}[m_e \bar{L}_e e + m_\mu \bar{L}_\mu \mu + m_\tau \bar{L}_\tau \tau]\phi_0$$

$$+v^{-1}[m_\tau \bar{L}_\mu \tau_R + m_\mu \bar{L}_e \mu_R + m_e \bar{L}_\tau e_R]\phi_1 + H.c.$$
$$+v^{-1}[m_\tau \bar{L}_e \tau_R + m_\mu \bar{L}_\tau \mu_R + m_e \bar{L}_\mu e_R]\phi_2 + H.c.$$

As a result, the rare decays $\tau^+ \to \mu^+\mu^+e^-$ and $\tau^+ \to e^+e^+\mu^-$ are allowed, but no others. For example, $\mu \to e\gamma$ is forbidden. Here $\phi_1^0, \bar{\phi}_2^0 \sim \omega$, mixing to form mass eigenstates $\psi_{1,2}^0 = (\phi_1^0 \pm \bar{\phi}_2^0)/\sqrt{2}$. Using

$$B(\tau^+ \to \mu^+\mu^+e^-)/B(\tau \to \mu\nu\nu)$$
$$= 9m_\tau^2 m_\mu^2 (m_1^2 + m_2^2)^2 / m_1^4 m_2^4,$$

the experimental upper limit of 2.3×10^{-8} yields the bound $m_1 m_2/\sqrt{m_1^2 + m_2^2} > 22$ GeV (174 Gev/v). Hence the production of $\psi_{1,2}^0 \bar{\psi}_{2,1}^0$ by Z' decay at the LHC with final states $\tau^-e^+\tau^-\mu^+$ and $\tau^+\mu^-\tau^+e^-$ would be indicative of this Z_3 flavor symmetry.

Cao/Damanik/Ma/Wegman(PRD 83, 093012 (2011)): Another possible way of producing $\psi_{1,2}$ is through the decay of a standard-model-like Higgs boson. This can happen in a model with 4 Higgs doublets, one singlet and one triplet under A_4, T_7, or $\Delta(27)$. The Higgs potential is different in each case, but there are common terms, and a simplified version has been analyzed. Here, there is a H^0 which couples to quarks with $\sqrt{2}$ times the standard-model coupling. It also couples to $\psi_{1,2}\bar{\psi}_{1,2}$. Since Z couples to $\psi_{1,2}\bar{\psi}_{2,1}$, this requires $m_1 + m_2 > 209$ GeV to respect LEPII data.

Observable Scalars at the LHC

The $\phi_{1,2}$ scalar doublets have $B - L = 0$, so they do not couple directly to the Z'_{B-L} gauge boson, but they can mix (after $U(1)_{B-L}$ breaking), with the χ and η singlets ($B - L = -2$) which do. Thus this model can be tested at the LHC by discovering Z' from $q\bar{q} \to Z' \to \mu^-\mu^+$ and looking for $Z' \to \psi_1\bar{\psi}_2 + \psi_2\bar{\psi}_1$ (assuming $m_1 = m_2$) with the subsequent decays $\psi \to \tau^- e^+$ and $\bar{\psi} \to \tau^-\mu^+$.
Let $\Gamma_0 = g^2_{B-L} m_{Z'}/12\pi$, then the partial decay widths of Z' are $\Gamma_q = (6)(3)(1/3)^2\Gamma_0$, $\Gamma_l = (3)(-1)^2\Gamma_0$, $\Gamma_\nu = (3)(-1)^2(1/2)\Gamma_0$, and $\Gamma_\psi \simeq (2)(-2)^2 \sin^4\theta(1/4)\Gamma_0$,

where $\sin\theta$ is an effective parameter accounting for the mixing of ψ to χ and η. The signature events are chosen to be $\tau^-\tau^-\mu^+e^+$ with τ^- decaying into ℓ^- (e^- or μ^-) plus missing energy. The background events yielding this signature come from

$$WWZ \;:\; pp \to W^+W^-Z,\; W^\pm \to \ell^\pm\nu,\; Z \to \ell^+\ell^-,$$
$$ZZ \;:\; pp \to ZZ,\; Z \to \ell^+\ell^-,\; Z \to \tau^+\tau^-,\; \tau^\pm \to \ell^\pm\nu\nu,$$
$$t\bar{t} \;:\; pp \to t\bar{t} \to b(\to \ell^-)\bar{b}(\to \ell^+)W^+W^-,\; W^\pm \to \ell^\pm\nu,$$
$$Zb\bar{b} \;:\; pp \to Zb(\to \ell^-)\bar{b}(\to \ell^+),\; Z \to \ell^+\ell^-.$$

We require no jet tagging and consider only events with both e^+ and μ^+ in the final states.

Our benchmark points for $m_{Z'}, m_\psi$ (in GeV) are (A) (1000,100), (B) (1500,100), (C) (1000,300), (D) (1500,300), with $g_{B-L} = e/\sin\theta_W$, and $\sin^2\theta = 0.2$.

We impose the following basic acceptance cuts:

$p_{T,\ell}^{(1,2)} > 50 \text{ GeV}, \;\; p_{T,\ell}^{(3,4)} > 20 \text{ GeV}, \;\; |\eta_\ell| < 2.5$,
$\Delta R_{\ell\ell'} > 0.4, \;\; \text{missing } E_T > 30 \text{ GeV}$,

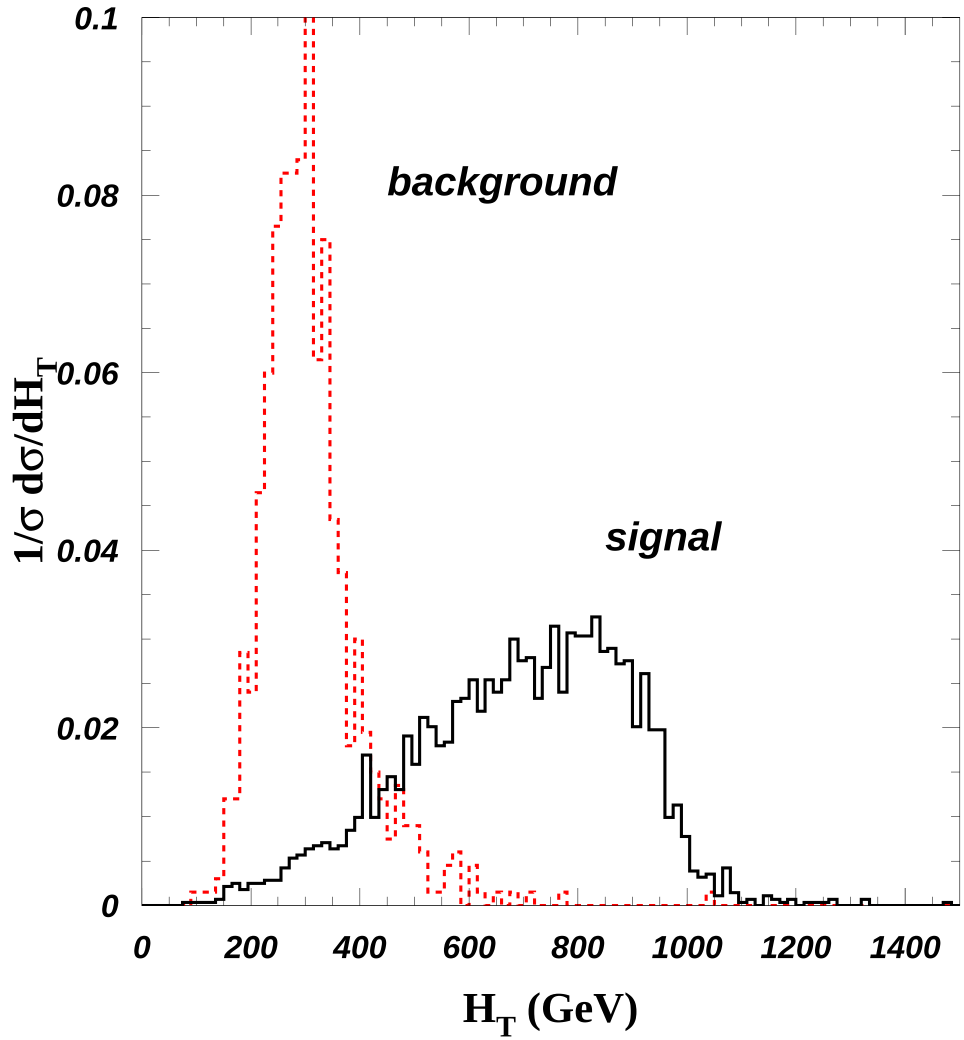
where ΔR_{ij} is the separation in the azimuthal angle (ϕ) - pseudorapidity (η) plane between i and j, defined as
$\Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$.
We also model detector resolution effects by smearing the final-state energy.

To further suppress the backgrounds, we require $H_T \equiv \sum_i p_{T,i} + \text{missing } E_T > 300 \text{ GeV}$, where i denotes the visible particles.

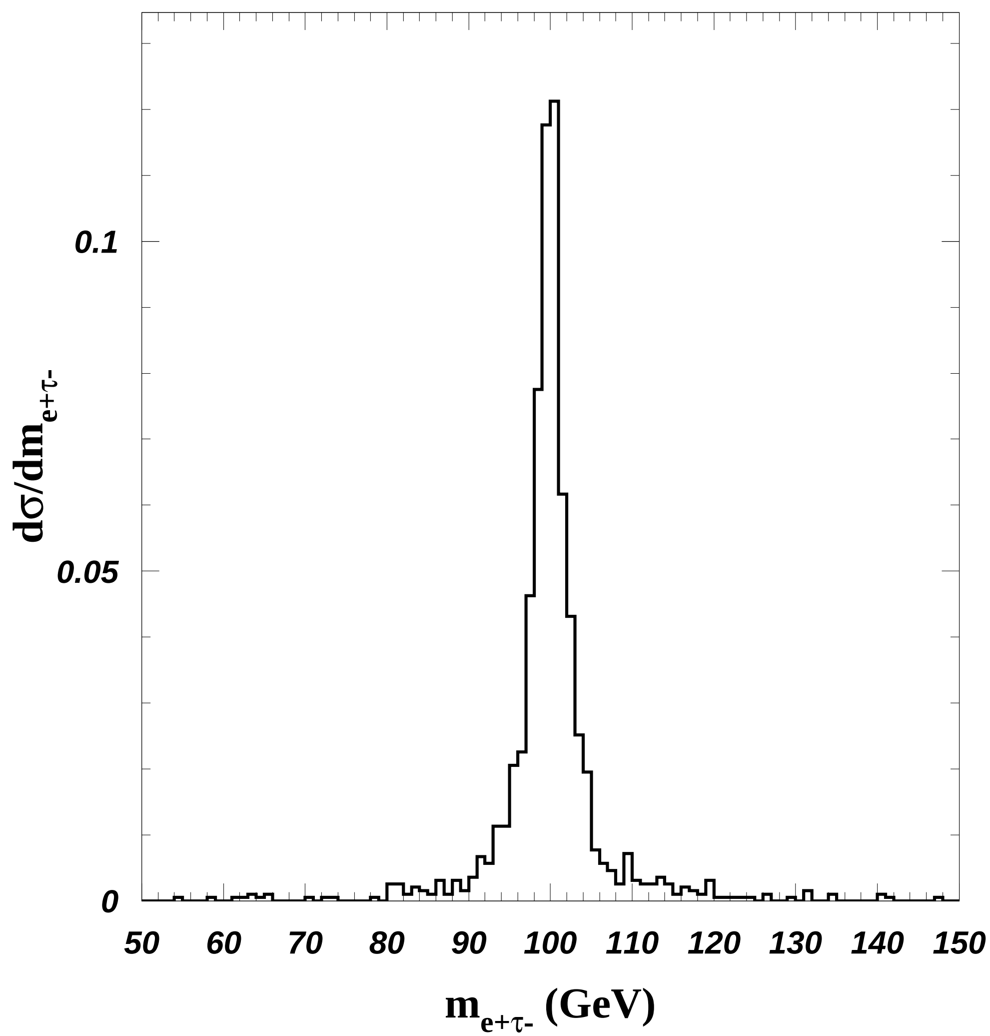
To reconstruct the scalar ψ, we adopt the collinear approximation that the ℓ and ν's from τ decays are parallel due to the τ's large boost, coming from the heavy ψ. Denoting by x_{τ_i} the fraction of the parent τ energy which each observable decay particle carries, the transverse momentum vectors are related by $\text{missing } E_T = (1/x_{\tau_1} - 1)\vec{p}_1 + (1/x_{\tau_2} - 1)\vec{p}_2$.
When the decay products are not back-to-back, this

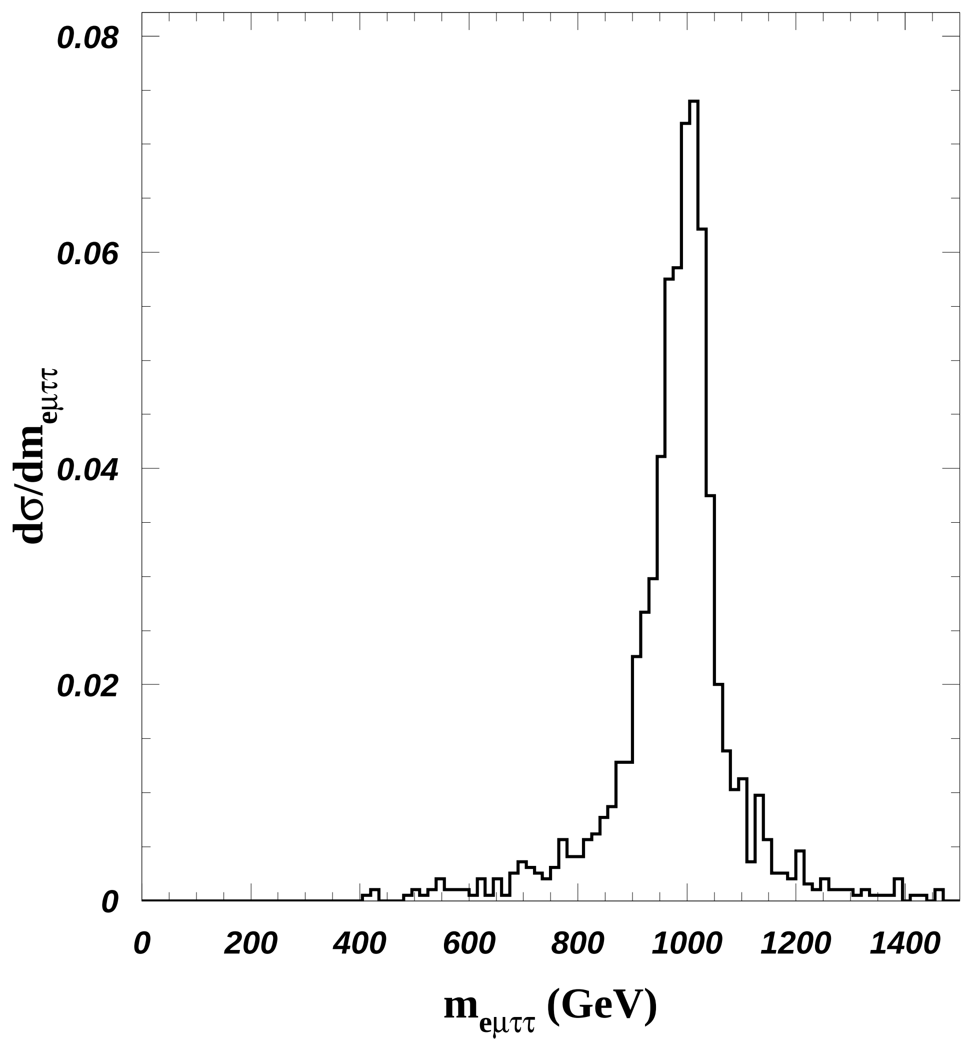
gives two conditions for x_{τ_i}, with the τ momenta as $\vec{p}_1/x_{\tau_1}$ and $\vec{p}_2/x_{\tau_2}$, respectively. We further require $x_{\tau_i} > 0$ to remove the unphysical solutions, and minimize $\Delta R_{e^+\ell^-}$ to choose the correct $e^+\ell^-$ to reconstruct ψ and then Z'. We then obtain the following signal and background cross sections (fb) at $E_{cm} = 14$ TeV:

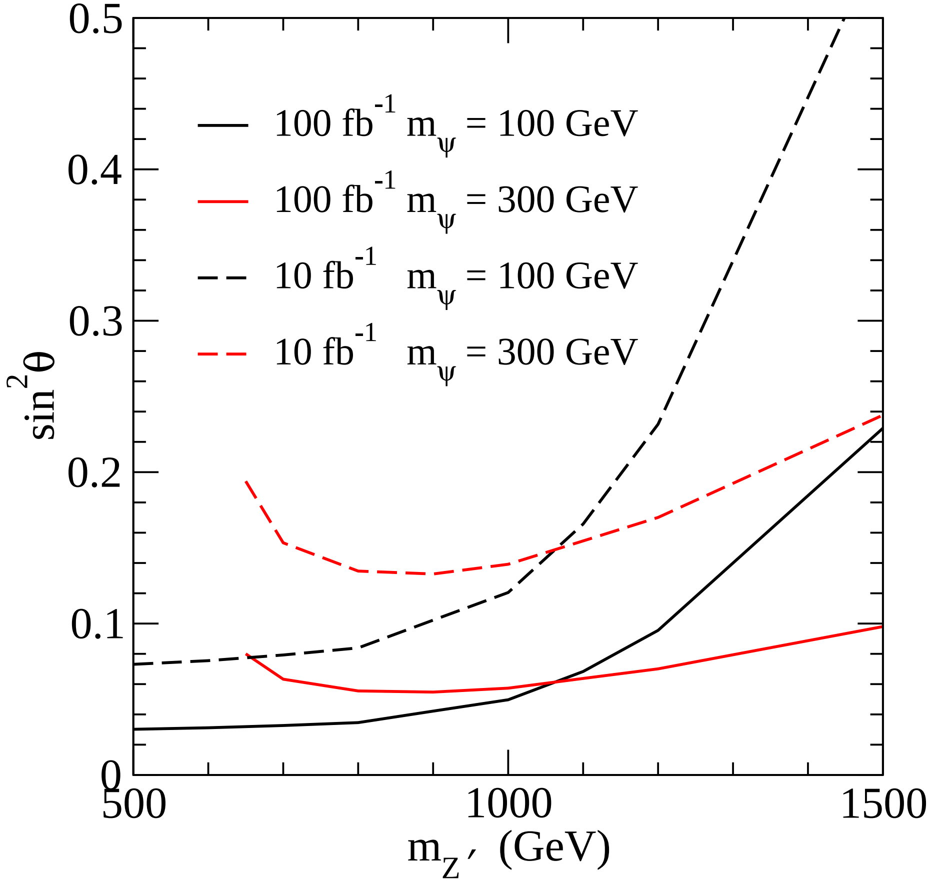
	(A)	(C)	$t\bar{t}$	WWZ	ZZ	$Zb\bar{b}$
no cut	5.14	2.57	1.22	0.21	27.11	2.99
basic cut	1.46	1.05	0.16	0.02	0.0052	0.024
H_T cut	1.41	1.04	0.08	0.006	0.0	0.0
$x_\tau > 0$	0.69	0.52	0.015	0.002	0.0	0.0



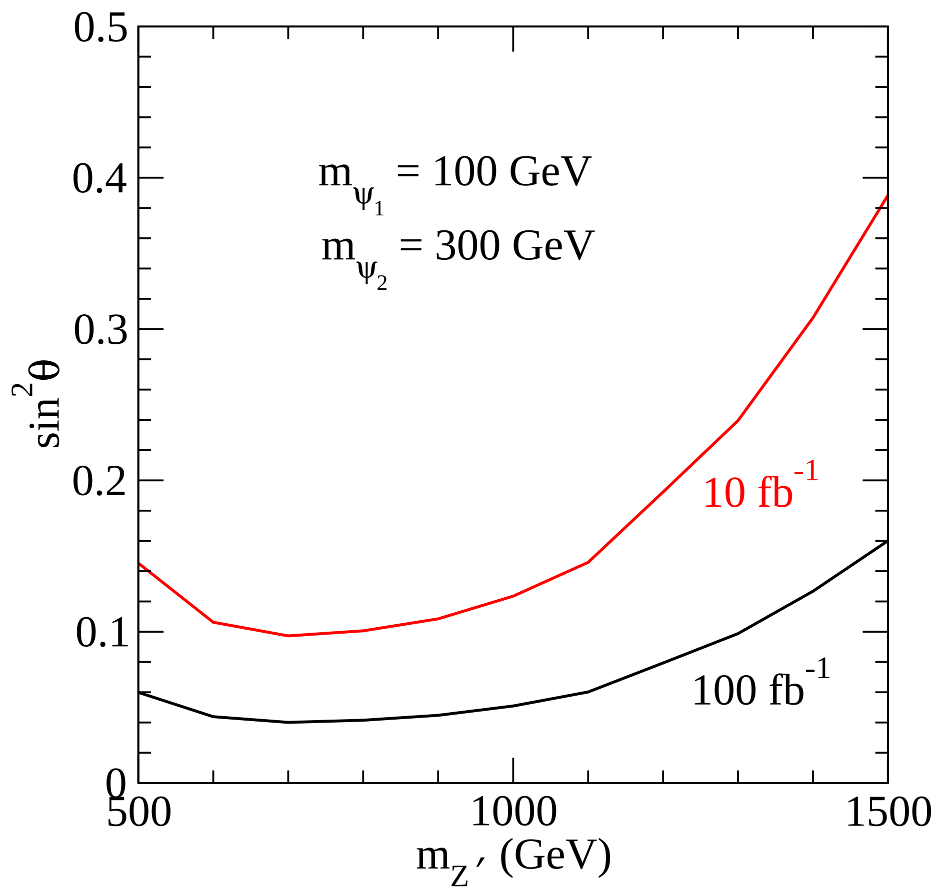
background
signal
1/σ dσ/dH_T
H_T (GeV)
0.1
0.08
0.06
0.04
0.02
0
0
200
400
600
800
1000
1200
1400


0.1
0.05
0
50
60
70
80
90
100
110
120
130
140
150
$d\sigma/dm_{e+\tau-}$
$m_{e+\tau-}$ (GeV)


0.08
0.06
0.04
0.02
0
$d\sigma/dm_{e\mu\tau\tau}$
0
200
400
600
800
1000
1200
1400
$m_{e\mu\tau\tau}$ (GeV)



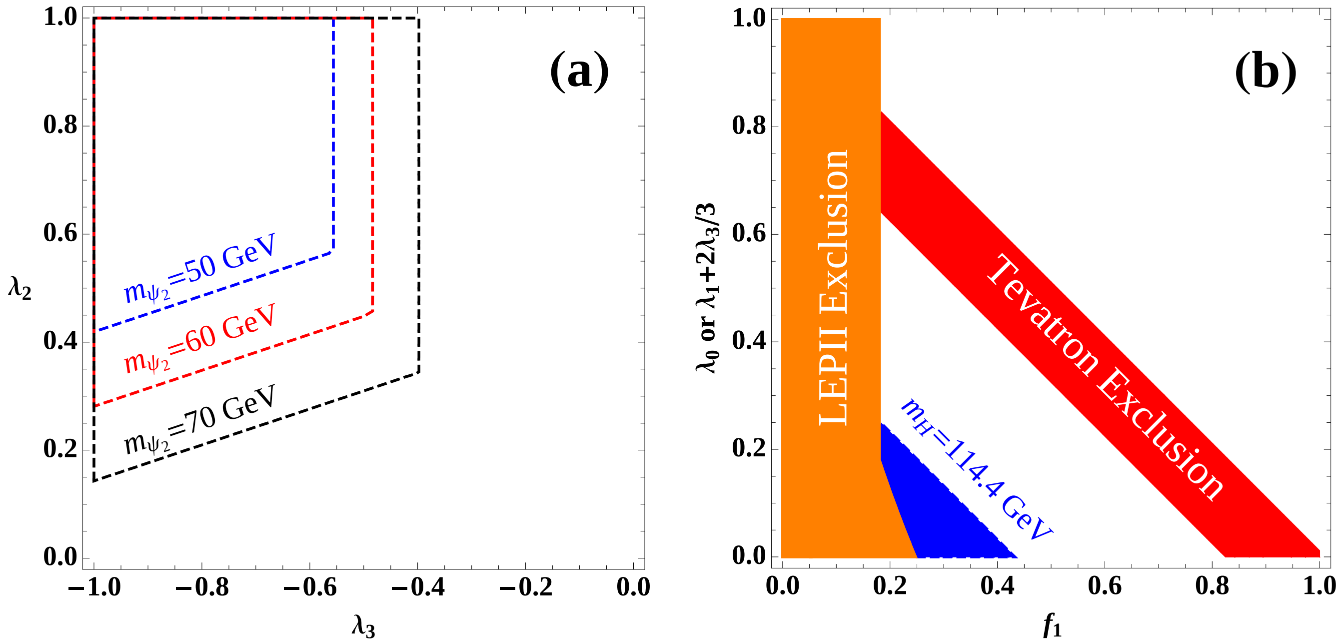
Discovery reach for $m_1 = m_2 = 100$ and 300 GeV at the LHC in the $m_{Z'} - \sin^2\theta$ plane.



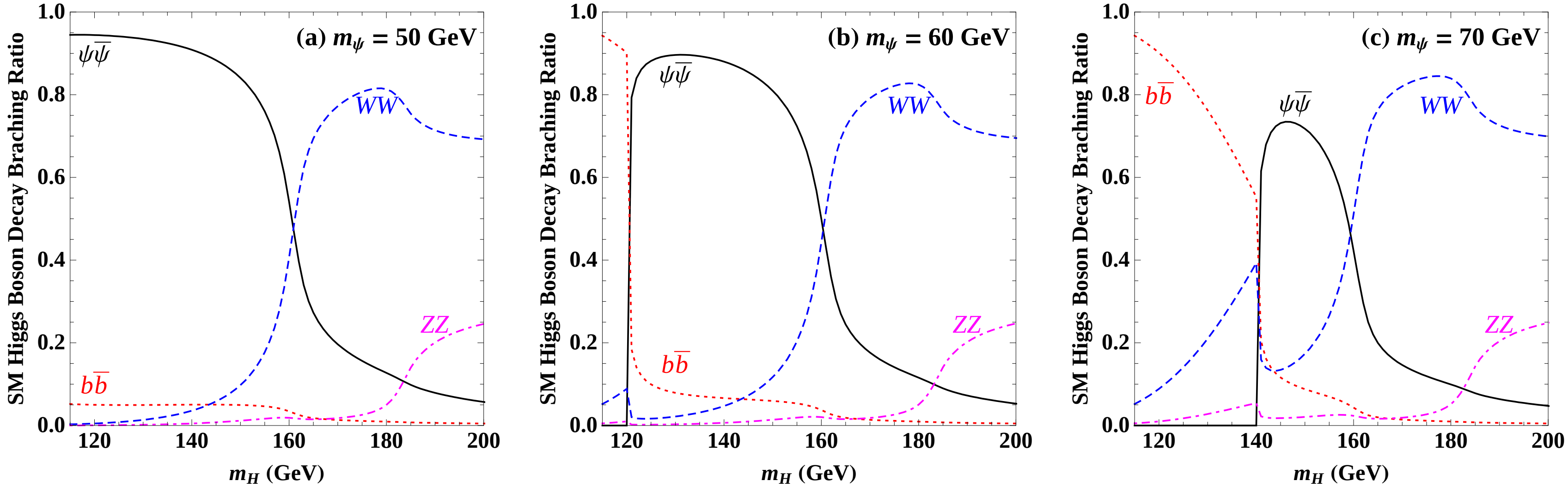
Discovery reach for $m_1 = 100$ and $m_2 = 300$ GeV at the LHC in the $m_{Z'} - \sin^2\theta$ plane.

Without Z', the production of H^0 and its decay into $\psi_2\bar{\psi}_2$ could also prove the existence of lepton triality. In the following, we assume $m_2 = 50$, 60, and 70 GeV and the decay $H^0 \to \psi_2\bar{\psi}_2$ with the subsequent decays $\psi_2 \to \tau^- e^+$ and $\bar{\psi}_2 \to \tau^- \mu^+$ at the LHC with $E_{cm} = 7$ TeV. Depending on the parameter f_1, this decay of H^0 could dominate over the standard-model channels, such as $b\bar{b}$ and maybe even W^+W^- and ZZ.

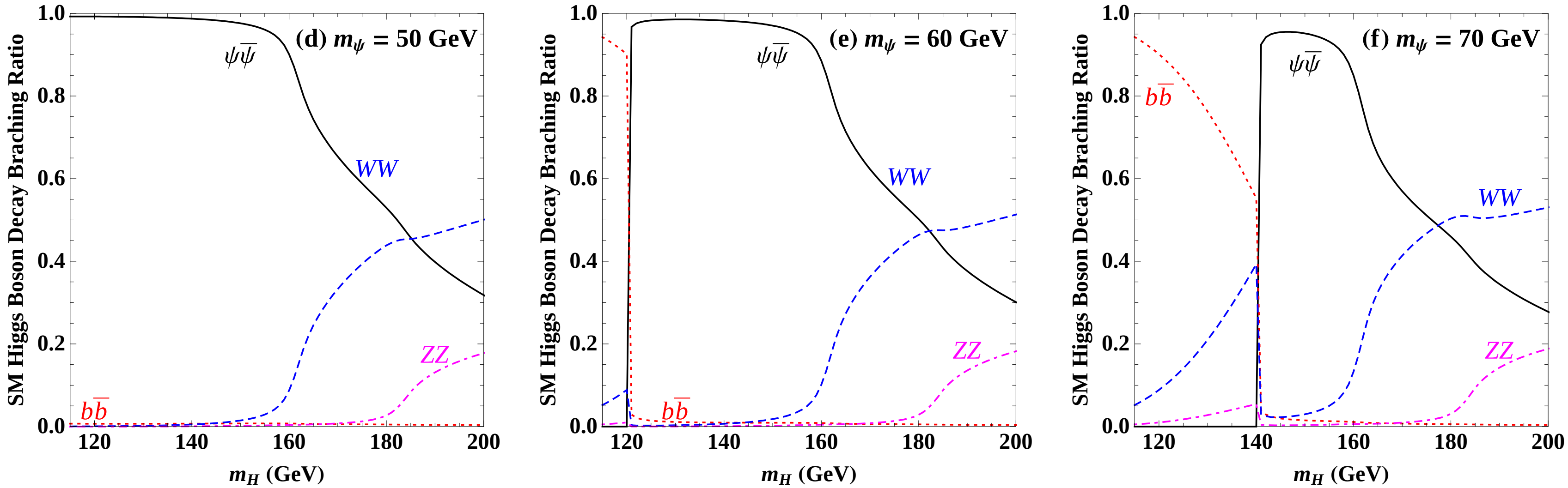
Using $f_1 = 0.5$ and $m_2 = 50(60)(70)$ GeV, we find for $m_H = 150$ GeV the cross sections 8.18 (13.42) (21.33) fb with total background of 0.2 fb from $t\bar{t}, ZZ, Zb\bar{b}, WWZ$.



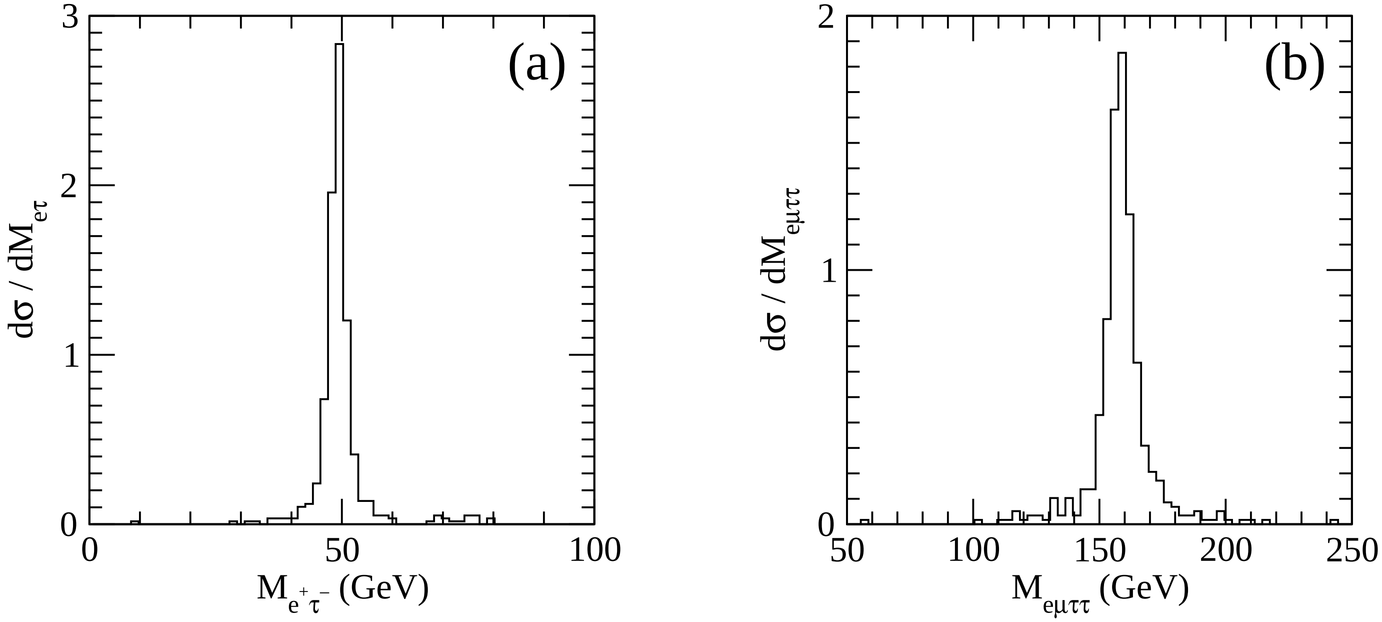
(a) Allowed region (inside each dashed box) in the plane of λ_2 and λ_3 for $m_2 =$ 50, 60, and 70 GeV. (b) Allowed region in th eplane of λ_0 (or $\lambda_1 + (2/3)\lambda_3$) and f_1.



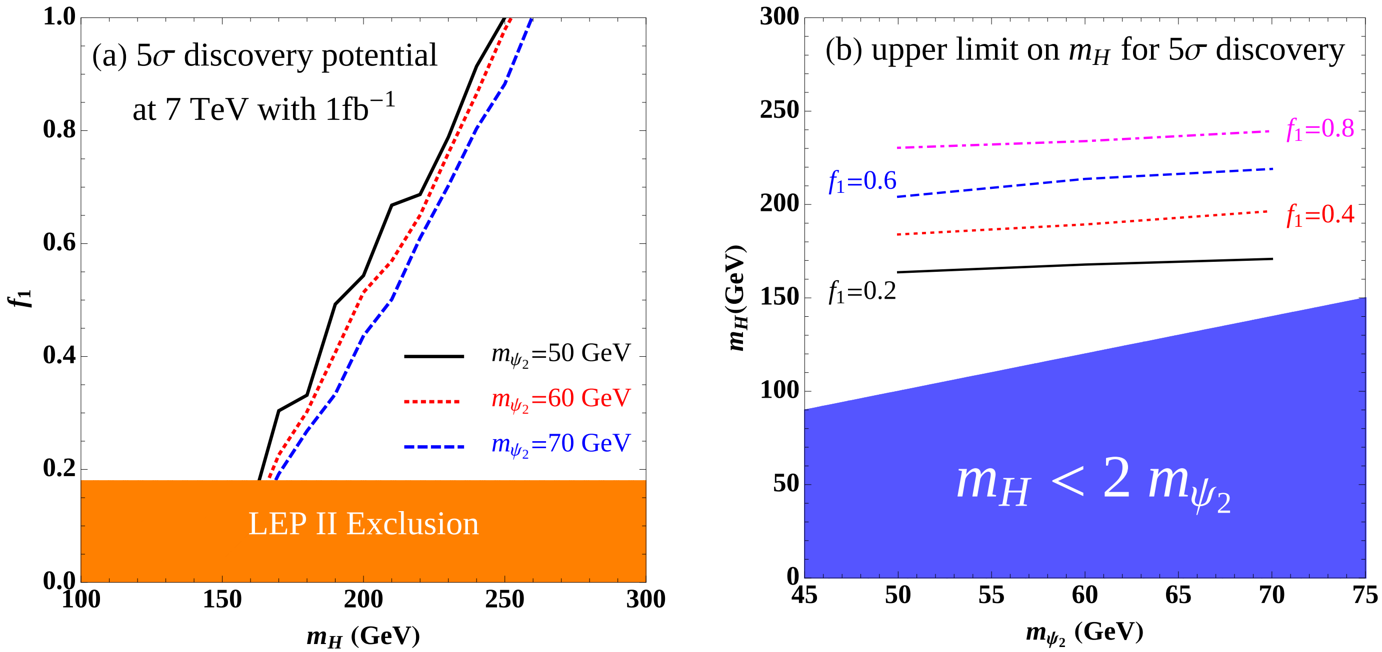
Decay branching fractions of H^0 versus m_H for $m_2 = 50$, 60, and 70 GeV with $f_1 = 0.18$.



Decay branching fractions of H^0 versus m_H for $m_2 = 50$, 60, and 70 GeV with $f_1 = 0.5$.



Reconstructed (a) m_2 from $e^+\tau^-$ and (b) m_H from $e^+\mu^+\tau^-\tau^-$ for $m_2 = 50$ GeV and $m_H = 160$ GeV.



Discovery reach for $m_2 = 50$, 60, and 70 GeV as a function of m_H at the LHC ($E_{cm} = 7$ TeV) with 1 fb^{-1}.

Conclusion

The application of $A_4, T_7, \Delta(27)$, etc. to explaining the neutrino mass matrix leads to the notion of lepton triality in the charged-lepton Yukawa sector. The physical neutral scalars $\psi_{1,2}$ decay into $\tau^- e^+$ or $\tau^+ \mu^-$.
In the case T_7 with gauged $B - L$, the Z' of this model may decay into $\psi_{1,2}\bar{\psi}_{2,1}$ through mixing and be observed at the LHC ($E_{cm} = 14$ TeV) with 10 fb^{-1}.
In a model with 4 Higgs doublets, the production and decay of a standard-model like H^0 into $\psi_2\bar{\psi}_2$ is also observable at the LHC ($E_{cm} = 7$ TeV) with 1 fb^{-1}.