Vacuum Effects in the 2HDM

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Based on JHEP08 (2011) 020 (73 pages), with R. Battye and G. Brawn

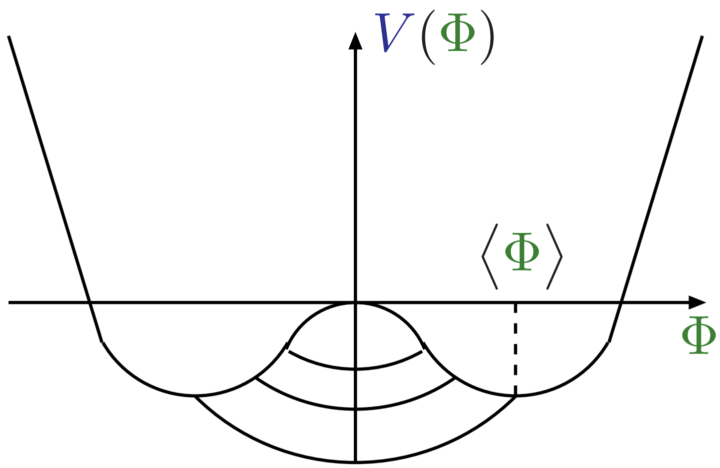
Plan of the talk

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The Two Higgs Doublet Model (2HDM) Potential
- The Majorana Formalism for the 2HDM Potential
- Symmetries of the 2HDM Potential
- Topological Defects in the 2HDM
- Conclusions

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in the SM: $SU(3)_{\rm colour} \otimes SU(2)_L \otimes U(1)_Y$

[P. W. Higgs '64; F. Englert, R. Brout '64.]



Higgs potential $V(\Phi)$

$$V(\Phi) \ = \ -m^2\Phi^\dagger\Phi \ + \ \lambda(\Phi^\dagger\Phi)^2 \,.$$

Ground state:

$$\langle\Phi\rangle \ = \ \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

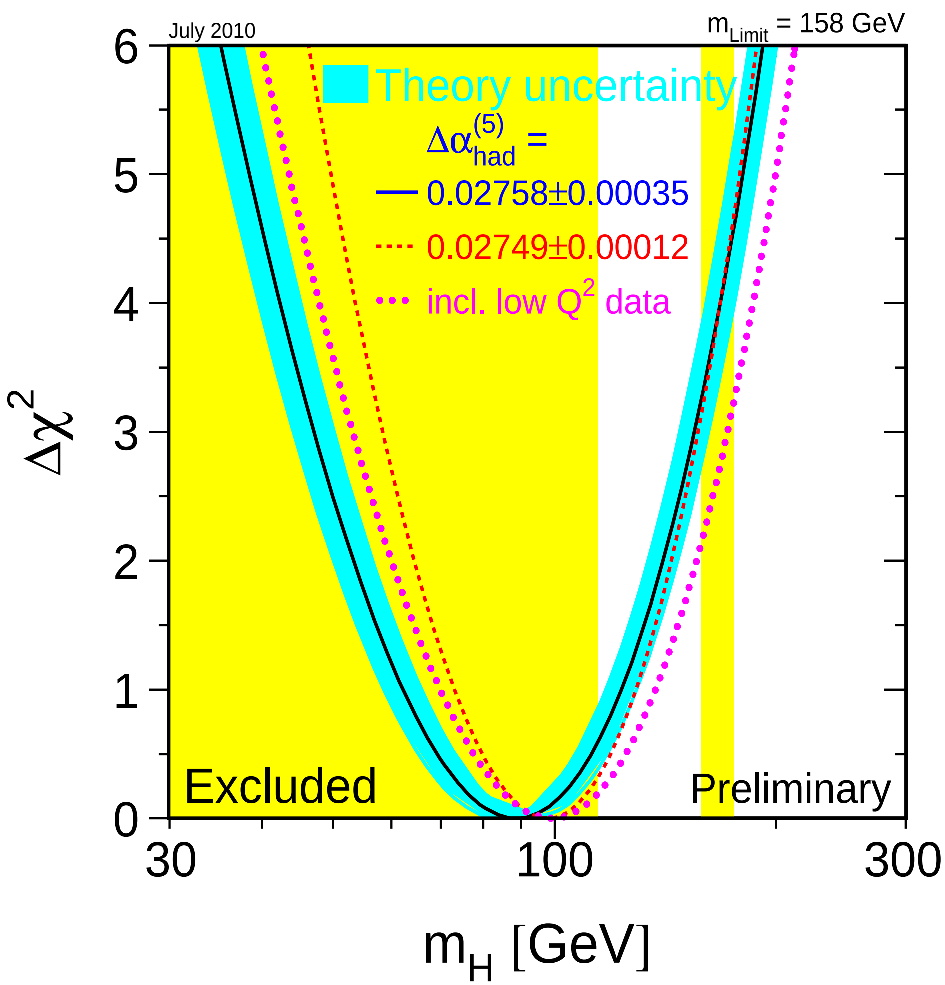
carries weak charge, but no electric charge and colour.

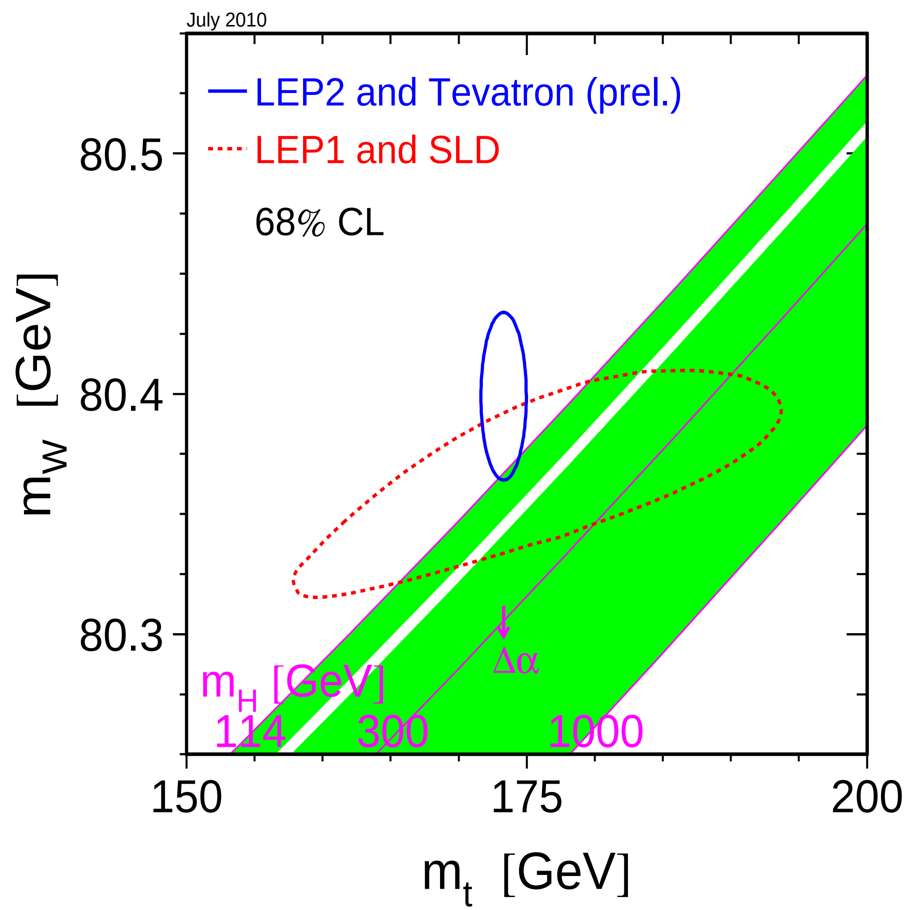
After Spontaneous Symmetry Breaking:

$\Rightarrow$ $W^\pm$, Z bosons and matter feel the presence of $\langle\Phi\rangle$ and become massive, but not γ and g^a, e.g. $M_W = g_w\langle\Phi\rangle$

$\Rightarrow$ Quantum excitations of $\Phi = \langle\Phi\rangle + H \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; H is the Higgs boson.

Light SM Higgs boson experimentally favourable





[LEP–TEVATRON EWG, `http://lepewwg.web.cern.ch/LEPEWWG/`]

• The 2HDM Potential

[T. D. Lee '73]

$$
\begin{aligned}
\mathrm{V} \; = \; & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)\; .
\end{aligned}
$$

V has 4 real mass parameters and 10 real quartic couplings.

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& + \lambda_1(\phi_1^\dagger\phi_1)^2 \;+\; \lambda_2(\phi_2^\dagger\phi_2)^2 \;+\; \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \;+\; \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 \;+\; \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \;+\; \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) \;+\; \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) \;+\; \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)\;.
\end{aligned}
$$

V has <u>4</u> real mass parameters and <u>10</u> real quartic couplings.

Remark on the *tree-level* **MSSM Higgs potential:**

$$
m_{12}^2 \;=\; -B\mu\,, \quad \lambda_1 \;=\; \lambda_2 \;=\; -\frac{1}{8}\,(g_w^2 + g'^2)\,, \quad \lambda_3 \;=\; -\frac{1}{4}\,(g_w^2 - g'^2)\,,
$$

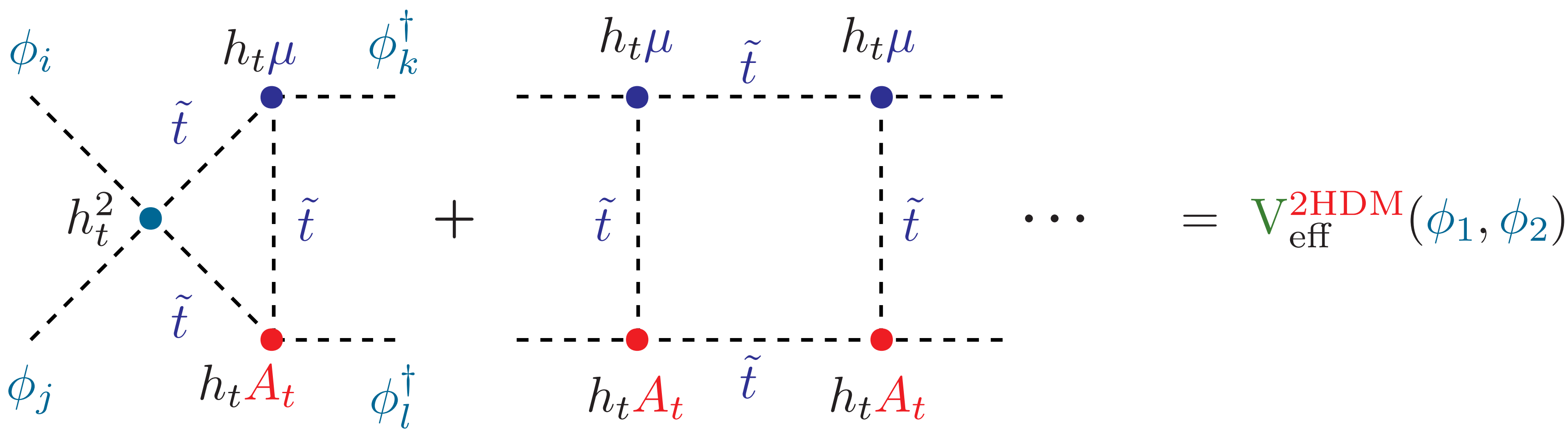
$$
\lambda_4 \;=\; \frac{1}{2}\,g_w^2\,, \qquad \lambda_5 \;=\; \lambda_6 \;=\; \lambda_7 \;=\; 0\,.
$$

$\phi_2 \to e^{i\arg m_{12}^2}\,\phi_2 \;\Longrightarrow\;$ Tree-level Higgs potential is <u>invariant</u> under CP

Beyond the *tree level*, **sizeable CP violation is induced in the Higgs sector through loop effects.**

Radiative Higgs-sector CP Violation:

[A.P. '98; A.P., C.E.M. Wagner '99; S. Y. Choi, M. Drees, J. S. Lee '00; M. Carena et al, '00 . . .]



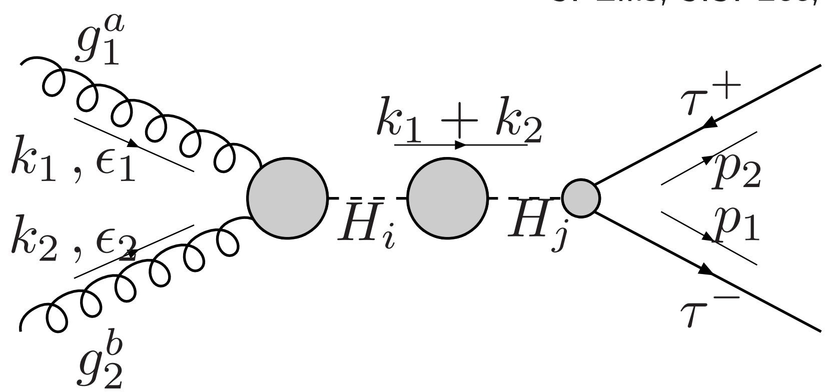
CP-violating terms are proportional to the rephasing invariant:

$$\boxed{\mathrm{Im}\,(\,m_{12}^{2*}\,\mu\,A_{t,b}\,)\;\neq\;0}$$

CP Violation in Higgs Production → Mixing → Decay at the LHC

[A.P., NPB504 (1997) 61;

J. Ellis, J.S. Lee, A.P., PRD70 (2004) 075010]



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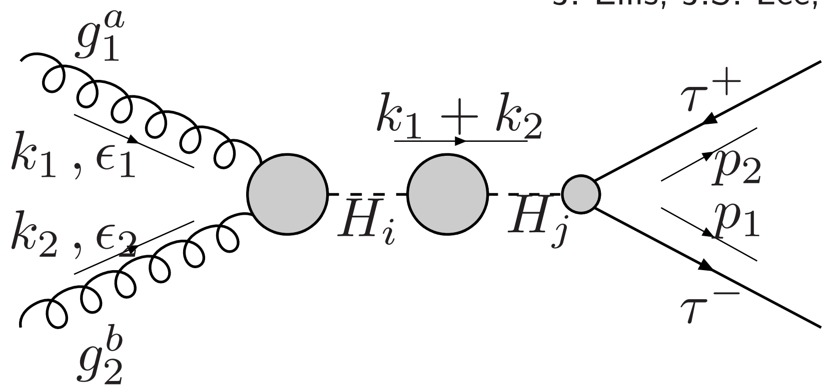
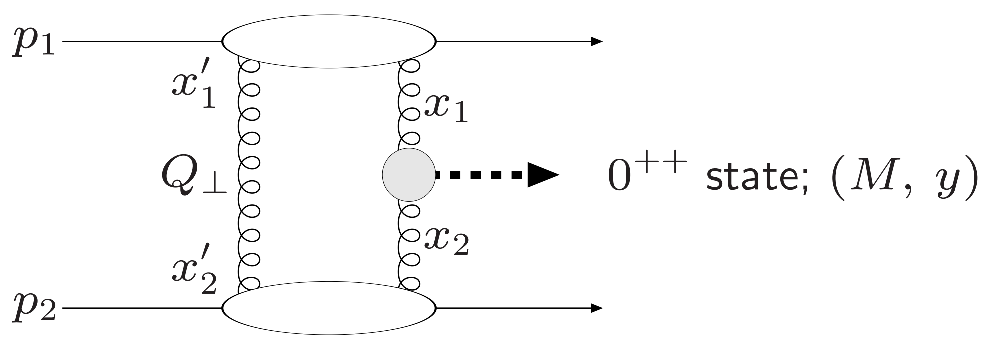
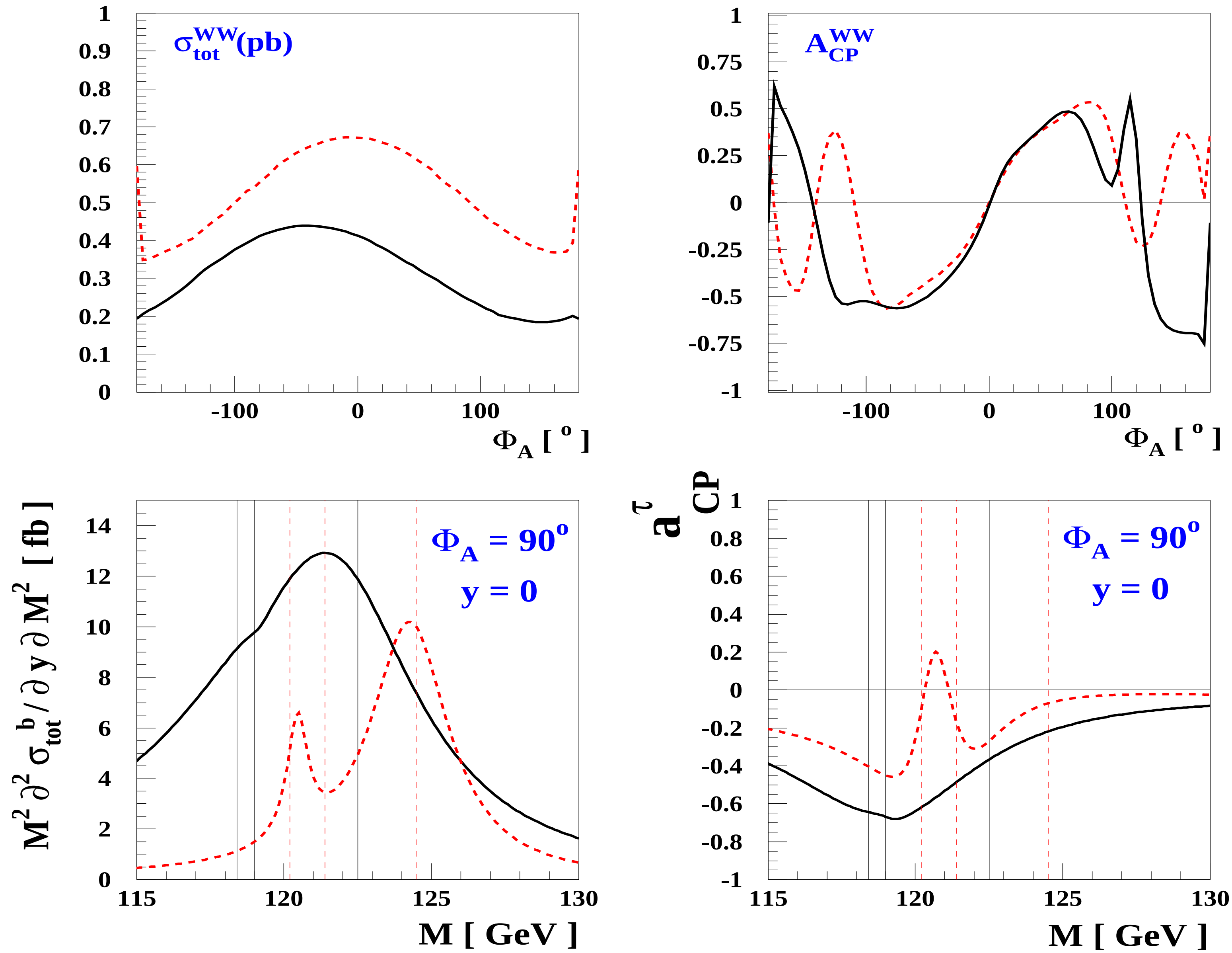


CP Violation in Diffractive Higgs Production at the LHC

[J. Ellis, J.S. Lee and A.P., PRD71 (2005) 075007.]



Resonant CP Violation at the LHC



[J. Ellis, J.S. Lee, A.P., PRD70 (2004) 075010; PRD71 (2005) 075007.]

• SO(1,3) Bilinear Scalar-Field Formalism

[C. C. Nishi '06; M. Maniatis et al '06; I. P. Ivanov '06]

Introduce the 4-vector:

$$\mathrm{r}^\mu \;\equiv\; \phi^\dagger \sigma^\mu \phi \;=\; \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ -i\left[\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1\right] \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \end{pmatrix}, \quad \text{with} \quad \phi \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

and σ^μ :

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Under a SL(2,**C**) reparameterization $\phi' \;=\; \mathrm{M}\,\phi$:

$$\mathrm{r}^\mu \quad\to\quad \mathrm{r}'^\mu \;=\; \Lambda^\mu_{\ \nu}\,\mathrm{r}^\nu\,,$$

with $\Lambda^\mu_{\ \nu} \in \mathrm{SO}(1,3)$.

References (*an incomplete list on symmetries in the 2HDM*)

- N. G. Deshpande and E. Ma, "Pattern Of Symmetry Breaking With Two Higgs Doublets," Phys. Rev. D **18** (1978) 2574.
- I. F. Ginzburg and M. Krawczyk, "Symmetries of two Higgs doublet model and CP violation," Phys. Rev. D **72** (2005) 115013.
- C. C. Nishi, "CP violation conditions in N-Higgs-doublet potentials," Phys. Rev. D **74** (2006) 036003 [Erratum-ibid. D **76** (2007) 119901].
- M. Maniatis, A. von Manteuffel, O. Nachtmann and F. Nagel, "Stability and symmetry breaking in the general two-Higgs-doublet model," Eur. Phys. J. C **48** (2006) 805.
- I. P. Ivanov, "Minkowski space structure of the Higgs potential in 2HDM," Phys. Rev. D **75** (2007) 035001 [Erratum-ibid. D **76** (2007) 039902].
- I. P. Ivanov, "Minkowski space structure of the Higgs potential in 2HDM: II. Minima, symmetries, and topology," Phys. Rev. D **77** (2008) 015017.
- P. M. Ferreira, H. E. Haber and J. P. Silva, "Generalized CP symmetries and special regions of parameter space in the two-Higgs-doublet model," Phys. Rev. D **79** (2009) 116004.
- *Review:* G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, "Theory and phenomenology of two-Higgs-doublet models," arXiv:1106.0034 [hep-ph].

• The 2HDM Potential in the SO(1,3) Formalism

$$V \;=\; -\,\frac{1}{2}\,\mathrm{M}_\mu \mathrm{r}^\mu \;+\; \frac{1}{4}\,\mathrm{L}_{\mu\nu}\mathrm{r}^\mu \mathrm{r}^\nu \;+\; \mathrm{V}_0\;,$$

with

$$\mathrm{M}_\mu \;=\; \Big(\; \mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2\;\Big)\;,$$

$$\mathrm{L}_{\mu\nu} \;=\; \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 \end{pmatrix}\,.$$

• **The 2HDM Potential in the SO(1,3) Formalism**

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Strengths: (i) Reduction of algebraic complexity of the 2HDM vacuum eqs.
(ii) Geometric understanding of conditions for CP invariance.

• The 2HDM Potential in the SO(1,3) Formalism

$$V \;=\; -\,\frac{1}{2}\,M_\mu r^\mu \;+\; \frac{1}{4}\,L_{\mu\nu} r^\mu r^\nu \;+\; V_0\;,$$

with

$$M_\mu \;=\; \Big(\; \mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2 \;\Big)\;,$$

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Strengths: (i) Reduction of algebraic complexity of the 2HDM vacuum eqs.
(ii) Geometric understanding of conditions for CP invariance.

Weaknesses: (i) Not **all** symmetries of the 2HDM potential are accounted for.
(ii) No covariant extension beyond the *tree-level* potential.

• Sufficient Conditions of Convexity for a *General* 2HDM Potential

V should be **bounded from below** for **all possible** $\phi_{1,2}$ directions at ∞, or equivalently for **all** $\mathrm{r}^{0,1,2,3} \to \infty \implies \mathrm{L}_{\mu\nu}$ should be **positive definite**.

<u>**4**</u> **constraints** derived from **Sylvester's criterion** for **positivity** of $\mathrm{L}_{\mu\nu}$:

$$
\begin{aligned}
\lambda_1 + \lambda_2 + \lambda_3 \quad &> \quad 0\,, \\
(\lambda_1 + \lambda_2 + \lambda_3)(\lambda_4 + R_5) - (R_6 + R_7)^2 \quad &> \quad 0\,, \\
(\lambda_1 + \lambda_2 + \lambda_3)(\lambda_4^2 - |\lambda_5|^2) - \lambda_4 \left[(R_6 + R_7)^2 + (I_6 + I_7)^2\right] & \\
-2I_5\,(R_6 + R_7)\,(I_6 + I_7) + R_5 \left[(R_6 + R_7)^2 - (I_6 + I_7)^2\right] \quad &> \quad 0\,, \\
\det\left(\mathrm{L}_{\mu\nu}\right) \quad &> \quad 0\,,
\end{aligned}
$$

with $R_k \equiv \mathrm{Re}\,\lambda_k$ and $I_k \equiv \mathrm{Im}\,\lambda_k$.

• The Majorana Formalism for the 2HDM Potential

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

• The Majorana Formalism for the 2HDM Potential

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Introduce the $\mathrm{SU(2)_L}$-covariant 8D complex field multiplet

$$\Phi \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix} ,$$

satisfying the **Majorana constraint**

$$\Phi \;=\; \mathrm{C}\,\Phi^* ,$$

where C is the **charge conjugation** 8D matrix

$$\mathrm{C} \;=\; \sigma^2 \otimes \sigma^0 \otimes \sigma^2 \;=\; \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & -i\,\sigma_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & -i\,\sigma_2 \\ i\,\sigma_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & i\,\sigma_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} .$$

• The SO(1,5) Bilinear Formalism

Introduce the 6-Vector

$$
\mathrm{R}^{\mathrm{A}} \;=\; \Phi^\dagger\, \Sigma^{\mathrm{A}}\, \Phi \;=\; \left(\begin{array}{c} \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\ \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\ -i \left[\phi_1^\dagger \phi_2 - \phi_2^\dagger \phi_1 \right] \\ \phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \\ \phi_1^{\mathsf{T}} i\sigma^2 \phi_2 - \phi_2^\dagger i\sigma^2 \phi_1^* \\ -i \left[\phi_1^{\mathsf{T}} i\sigma^2 \phi_2 + \phi_2^\dagger i\sigma^2 \phi_1^* \right] \end{array} \right) ,
$$

with $\mathrm{A} = \mu,\ 4,\ 5$ and

$$
\Sigma^\mu \;=\; \frac{1}{2} \left(\begin{array}{cc} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^{\mathsf{T}} \end{array} \right) \otimes \sigma^0 \,,
$$

$$
\Sigma^4 \;=\; \frac{1}{2} \left(\begin{array}{cc} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{array} \right) \otimes \sigma^0 \,, \qquad \Sigma^5 \;=\; \frac{1}{2} \left(\begin{array}{cc} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{array} \right) \otimes \sigma^0 \,.
$$

• **GL(4,C) Reparameterization of $\Phi \;\to\; \Phi' = \mathrm{M}\,\Phi$ (with $\mathrm{M}^* = \mathrm{C}^{-1}\mathrm{MC}$)**

$$\mathrm{R}^{\mathrm{A}} \quad \mapsto \quad \mathrm{R}'^{\,\mathrm{A}} \;=\; e^{\sigma/8}\,\Lambda^{\mathrm{A}}{}_{\mathrm{B}}\,\mathrm{R}^{\mathrm{B}}\;,$$

where $e^{\sigma} = \det[\mathrm{M}^{\dagger}\mathrm{M}] > 0$ and $\Lambda^{\mathrm{A}}{}_{\mathrm{B}} \in \mathrm{SO}(1,5)$.

• • •

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. . .

Majorana condition on R^{A}: $\quad \mathrm{R}^{\mathrm{A}} \ = \ \mathrm{R}^{\mathrm{A}}_{\mathrm{C}} \ \Longleftrightarrow \ \mathrm{C}^{-1}\,\Sigma^{\mathrm{A}}\,\mathrm{C} \ = \ (\Sigma^{\mathrm{A}})^{\mathsf{T}}$.

. . .

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...

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...

Clifford-like algebra:

$$\Sigma^{\mathrm{A}}\,\overline{\Sigma}^{\mathrm{B}} \;+\; \Sigma^{\mathrm{B}}\,\overline{\Sigma}^{\mathrm{A}} \;=\; \frac{1}{2}\,\eta^{\mathrm{AB}}\,\mathbf{I}_8\;,$$

with $\overline{\Sigma}^{\mathrm{A}} \equiv (\Sigma^0, -\Sigma^{1,2,3,4,5})$ and $\eta^{\mathrm{AB}} = \mathrm{diag}\,(1,-1,-1,-1,-1,-1)$.

...

• GL(4,C) Reparameterization of $\Phi \;\to\; \Phi' = \mathrm{M}\,\Phi$ (with $\mathrm{M}^* = \mathrm{C}^{-1}\mathrm{MC}$)

$$\mathrm{R}^{\mathrm{A}} \quad\mapsto\quad \mathrm{R}'^{\,\mathrm{A}} \;=\; e^{\sigma/8}\,\Lambda^{\mathrm{A}}{}_{\mathrm{B}}\,\mathrm{R}^{\mathrm{B}}\ ,$$

where $e^{\sigma} = \det[\mathrm{M}^{\dagger}\mathrm{M}] > 0$ and $\Lambda^{\mathrm{A}}{}_{\mathrm{B}} \in \mathrm{SO}(1,5)$.

$$\cdots$$

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$$\cdots$$

Clifford-like algebra:

$$\Sigma^{\mathrm{A}}\,\overline{\Sigma}^{\mathrm{B}} \;+\; \Sigma^{\mathrm{B}}\,\overline{\Sigma}^{\mathrm{A}} \;=\; \frac{1}{2}\,\eta^{\mathrm{AB}}\,\mathbf{I}_8\ ,$$

with $\overline{\Sigma}^{\mathrm{A}} \equiv (\Sigma^0, -\Sigma^{1,2,3,4,5})$ and $\eta^{\mathrm{AB}} = \mathrm{diag}\,(1,-1,-1,-1,-1,-1)$.

$$\cdots$$

Majorana-constrained U(4) rotations $\Phi \;\to\; \Phi' = \mathrm{U}\,\Phi$, with $\mathrm{U}^* = \mathrm{C}^{-1}\,\mathrm{U}\,\mathrm{C}$, induce SO(5) rotations on R^{I} (with $\mathrm{I}\,,\ \mathrm{J} = 1,2,3,4,5$):

$$\mathrm{R}^{\mathrm{I}} \quad\to\quad \mathrm{R}'^{\mathrm{I}} \;=\; \mathrm{O}^{\mathrm{I}}{}_{\mathrm{J}}\,\mathrm{R}^{\mathrm{J}}\ , \quad \text{with} \quad \mathrm{O} \in \mathrm{SO}(5)\ .$$

• Symmetries of the 2HDM Potential

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\mathrm{V} \;=\; -\,\frac{1}{2}\,\mathrm{M_A\,R^A} \;+\; \frac{1}{4}\,\mathrm{L_{AB}\,R^A\,R^B}\;.$$

$$\ldots$$

Classification of **all possible** proper, improper and semi-simple subgroups of **SO(5)**:

I. $\mathrm{SO(5)}$;

II. $\mathrm{O(4)\otimes Z_2}$; $\mathrm{SO(4)}$;

III. $\mathrm{O(3)\otimes O(2)}$; $\mathrm{SO(3)\otimes (Z_2)^2}$; $\mathrm{O(3)\otimes Z_2}$; $\mathrm{SO(3)}$;

IV. $\mathrm{O(2)\otimes O(2)\otimes Z_2}$; $\mathrm{O(2)\otimes O(2)}$; $\mathrm{O(2)\otimes (Z_2)^3}$; $\mathrm{SO(2)\otimes (Z_2)^2}$; $\mathrm{O(2)\otimes Z_2}$; $\mathrm{SO(2)}$;

V. $\mathrm{(Z_2)^4}$; $\mathrm{(Z_2)^2}$.

15 distinct symmetries, but **not all** are invariant under $\mathrm{U(1)_Y \simeq SO(2)}$.

• Symmetries of the $U(1)_Y$-Invariant 2HDM Potential

SO(5)-diagonally reduced basis: $\mathrm{Im}\,\lambda_5 = 0$ and $\lambda_6 = \lambda_7$.

The 2HDM potential exhibits a **total** of **13** $=$ 6 $+$ 7 accidental Higgs-Family (HF) and CP symmetries:

Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\lambda_6 = \lambda_7$
$(Z_2)^2 \times SO(2)$	–	–	0	–	–	–	–	–	0
$O(2) \times O(2)$	–	–	0	–	–	–	–	0	0
$O(3) \times O(2)$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0
$Z_2 \times O(2)$	–	–	Real	–	–	–	–	–	Real
$(Z_2)^3 \times O(2)$	–	μ_1^2	0	–	λ_1	–	–	–	0
$Z_2 \times [O(2)]^2$	–	μ_1^2	0	–	λ_1	–	–	$2\lambda_1 - \lambda_{34}$	0
SO(5)	–	μ_1^2	0	–	λ_1	$2\lambda_1$	0	0	0
$Z_2 \times O(4)$	–	μ_1^2	0	–	λ_1	–	0	0	0
$SO(4)$	–	–	0	–	–	–	0	0	0
$O(2) \times O(3)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	–	0	0
$(Z_2)^2 \times SO(3)$	–	μ_1^2	0	–	λ_1	–	–	λ_4	0
$Z_2 \times O(3)$	–	μ_1^2	Real	–	λ_1	–	–	λ_4	Real
$SO(3)$	–	–	Real	–	–	–	–	λ_4	Real

• **Focus on the 6 generic HF/CP symmetries of the 2HDM Potential**

Symmetry	Frequent Name	HF transormation in $(\phi_1\,,\ \phi_2)$ Basis	CP transformation in $(\phi_1\,,\ \phi_2)$ Basis
$(\mathrm{Z}_2)^2 \times \mathrm{SO}(2)$	Z_2	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	—
$\mathrm{O}(2) \times \mathrm{O}(2)$	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$ $\alpha \in [0, \pi)$	—
$\mathrm{O}(3) \times \mathrm{O}(2)$	$\mathrm{SO}(3)_{\mathrm{HF}}$	$\begin{pmatrix} e^{-i\alpha}\cos\theta & e^{-i\beta}\sin\theta \\ -e^{i\beta}\sin\theta & e^{i\alpha}\cos\theta \end{pmatrix}$ $\theta, \alpha, \beta \in [0, \pi)$	—
$\mathrm{Z}_2 \times \mathrm{O}(2)$	CP1	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$(\mathrm{Z}_2)^3 \times \mathrm{O}(2)$	CP2	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$\mathrm{Z}_2 \times [\mathrm{O}(2)]^2$	CP3	$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ $\theta \in [0, \pi)$	$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ $\theta \in [0, \pi)$

• Topological Defects in the 2HDM

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Symmetry	$\mathrm{G}_{\mathrm{HF/CP}}$	$\mathrm{H}_{\mathrm{HF/CP}}$	$\mathcal{M}_{\Phi}^{\mathrm{HF/CP}}$	Topological Defect
Z_2	Z_2	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{U(1)}_{\mathrm{PQ}}$	$\mathrm{U(1)}_{\mathrm{PQ}} \simeq S^1$	$\mathbf{I}$	S^1	Vortex
$\mathrm{SO(3)}_{\mathrm{HF}}$	$\mathrm{SO(3)}_{\mathrm{HF}}$	$\mathrm{SO(2)}_{\mathrm{HF}}$	S^2	Global Monopole
CP1	$\mathrm{CP1} \simeq \mathrm{Z}_2$	$\mathbf{I}$	Z_2	Domain Wall
CP2	$\mathrm{Z}_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
CP3	$\mathrm{CP1} \otimes \mathrm{SO(2)}$	CP1	S^1	Vortex

• **Energy density of the topological defect $\phi_{1,2}(\mathbf{r})$:**

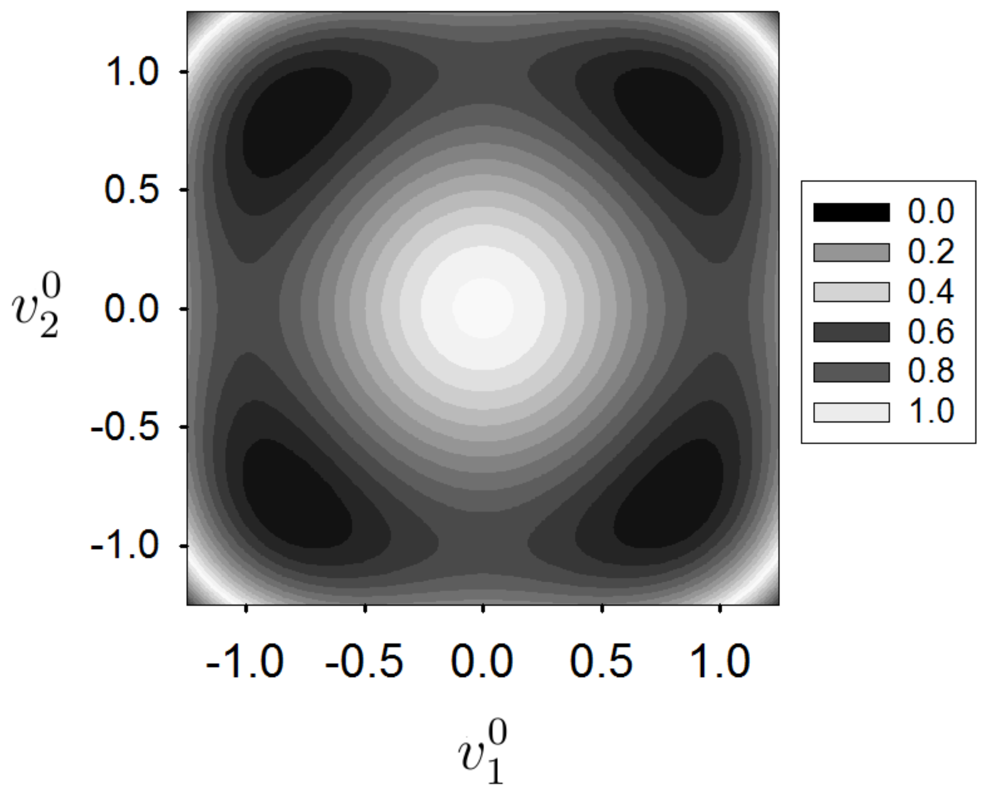
$$\mathcal{E}(\phi_1, \phi_2) \; = \; (\nabla\phi_1^\dagger)\cdot(\nabla\phi_1) + (\nabla\phi_2^\dagger)\cdot(\nabla\phi_2) + \; \mathrm{V}(\phi_1,\phi_2) \; + \; \mathrm{V}_0 \; .$$

• **Gradient flow numerical approach to find $\phi_{1,2}(\mathbf{r})$**

$$-\frac{\delta E[\phi_{1,2}]}{\delta\phi_{1,2}(\mathbf{r},\tau)} \; = \; \frac{\partial\phi_{1,2}(\mathbf{r},\tau)}{\partial\tau} \; \to \; 0 \; , \quad \text{for } \tau \gg 1 \; .$$

• Z_2 Domain Walls

$$\begin{array}{ccc}
\begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} & \xleftrightarrow{\mathrm{U(1)_Y}} & \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix} \\
\mathrm{Z}_2 \updownarrow & & \updownarrow \mathrm{Z}_2 \\
\begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} & \xleftrightarrow{\mathrm{U(1)_Y}} & \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}
\end{array}$$

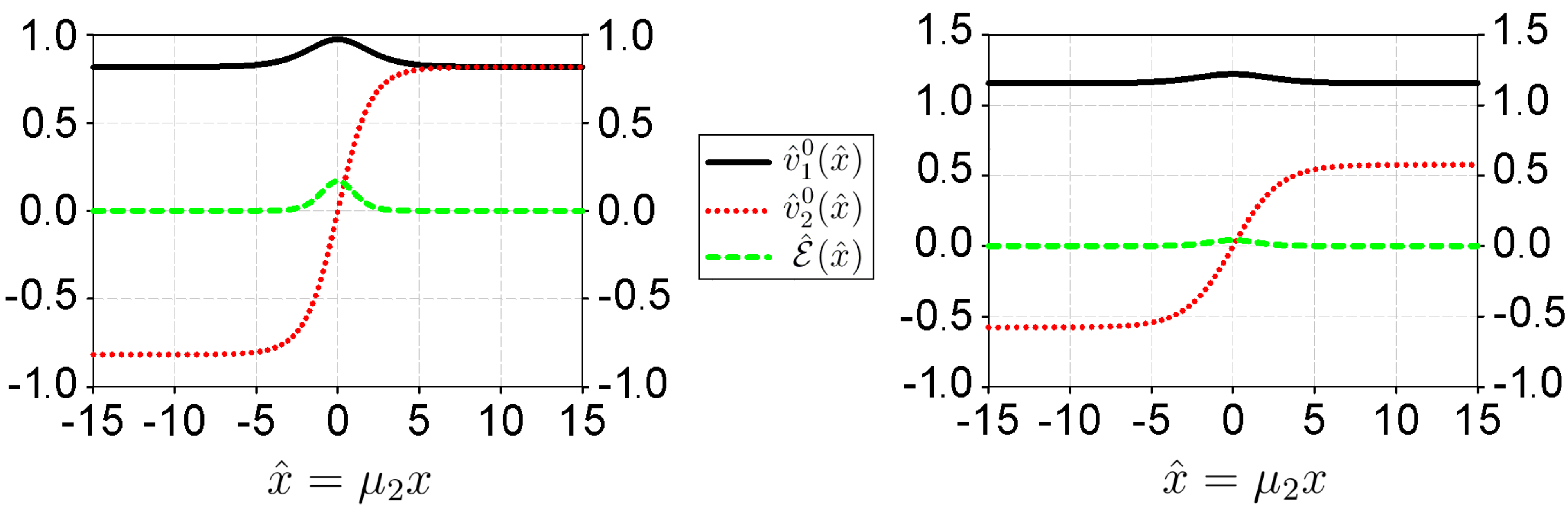


Spatial profile of the $\mathbf{Z}_2$ domain wall

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Introduce dimensionless quantities:

$$\hat{x} \;=\; \mu_2 x\;, \quad \hat{v}^0_{1,2}(\hat{x}) \;=\; \frac{v^0_{1,2}(\hat{x})}{\eta}\;, \quad \hat{\mathrm{E}} \;=\; \frac{\lambda_2 \mathrm{E}}{\mu_2^3}\;, \quad \text{with } \eta \;=\; \frac{\mu_2}{\sqrt{\lambda_2}}\;.$$

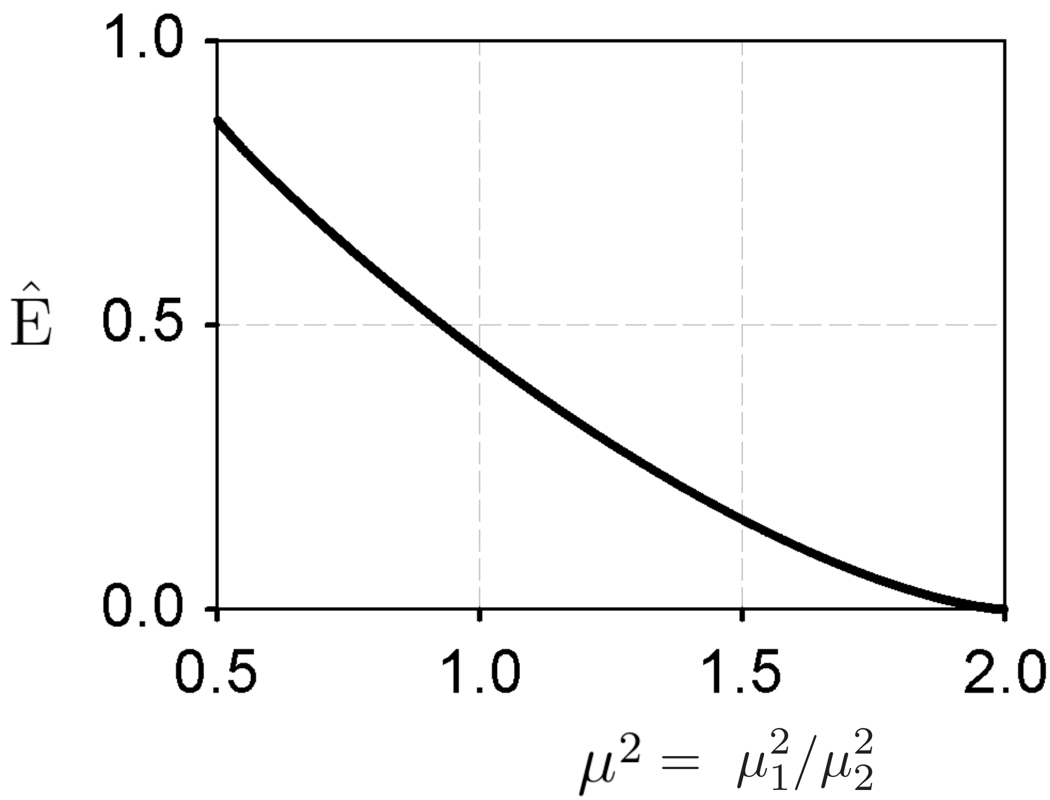


Energy dependence of the $\mathbf{Z}_2$ domain wall

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Energy per unit area:

$$\mathrm{E} \;=\; \int_{-\infty}^{\infty} dx \;\; \mathcal{E}(\phi_1, \phi_2) \;\; .$$

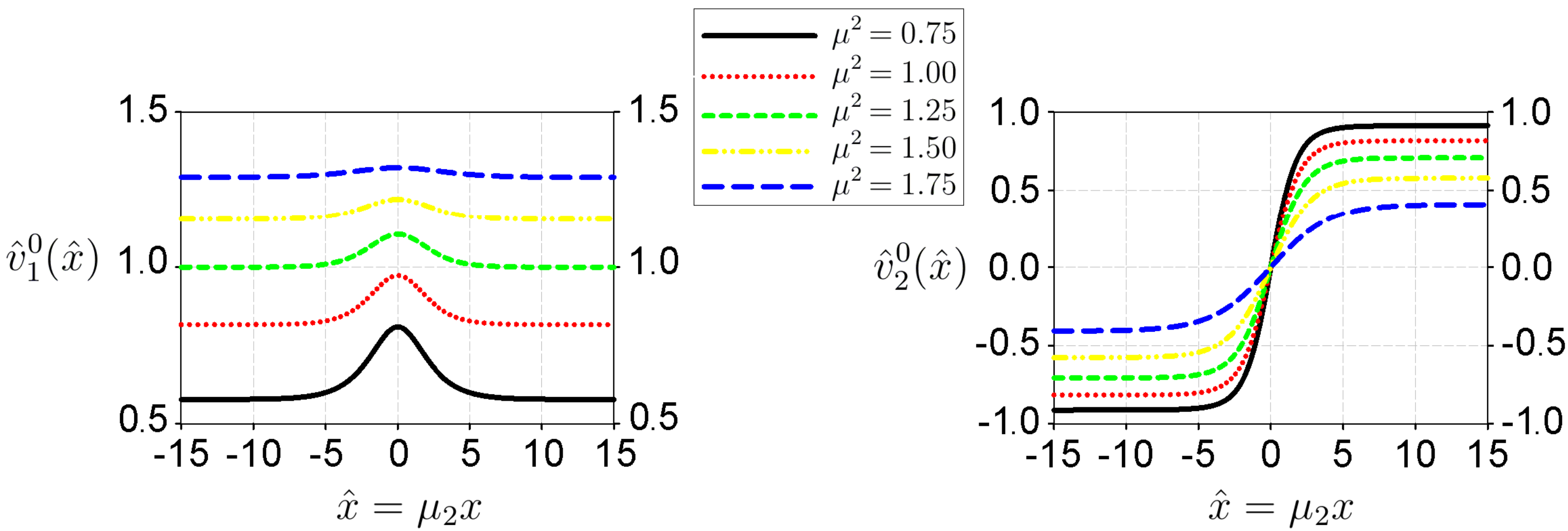


Comparison of spatial profiles of $\mathbf{Z}_2$ domain walls

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Introduce the dimensionless ratio:

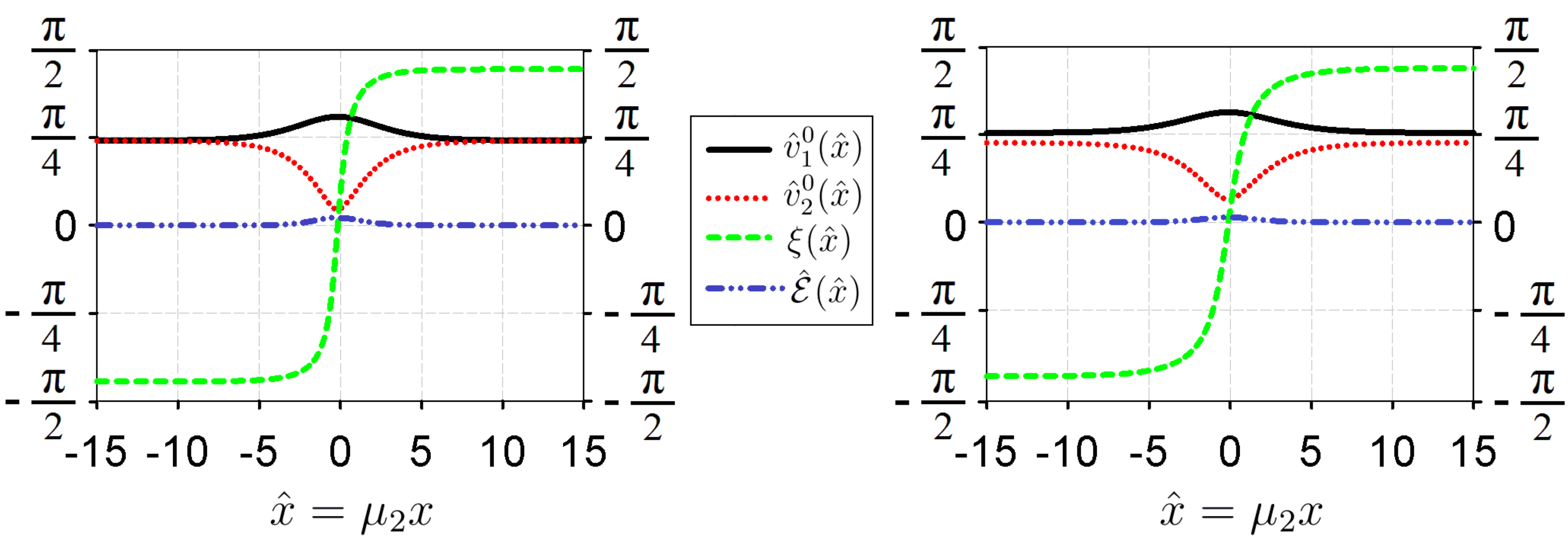
$$\mu^2 \;=\; \frac{\mu_1^2}{\mu_2^2}\,.$$



• CP1 Domain Walls

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

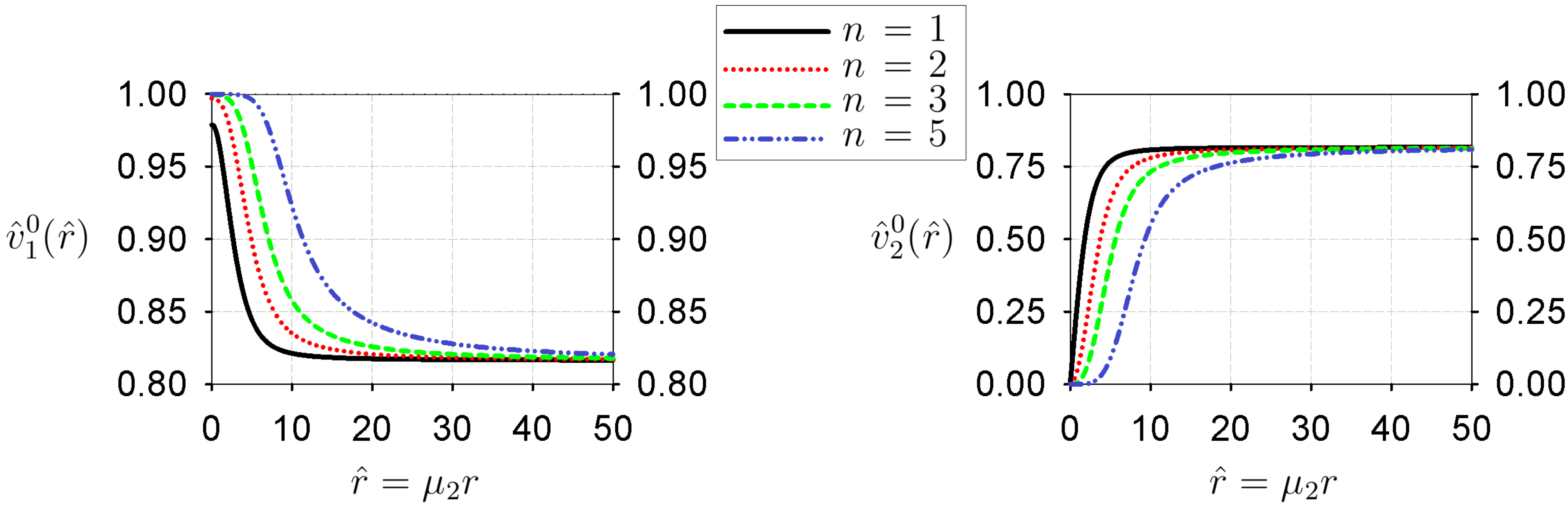
$$\phi_1(x) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1^0(x) \end{pmatrix}, \qquad \phi_2(x) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2^0(x)\, e^{i\xi(x)} \end{pmatrix}.$$



• $\mathbf{U(1)}_{\mathrm{PQ}}$ **Vortices**

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2^0(r) e^{in\chi} \end{pmatrix}.$$

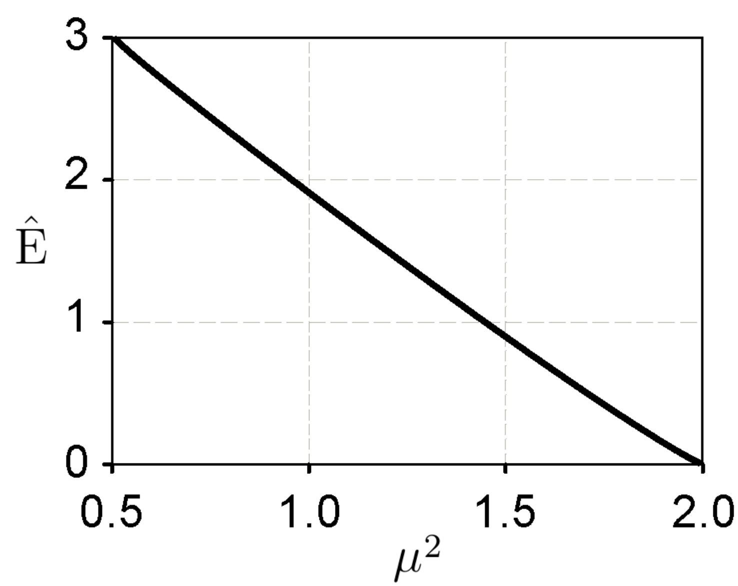


Energy dependence of the $\mathbf{U(1)}_{\mathrm{PQ}}$ Vortex

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Energy per unit length:

$$\mathrm{E} \;=\; 2\pi \int_0^\infty r dr \;\; \mathcal{E}(\phi_1, \phi_2) \;\; ,$$

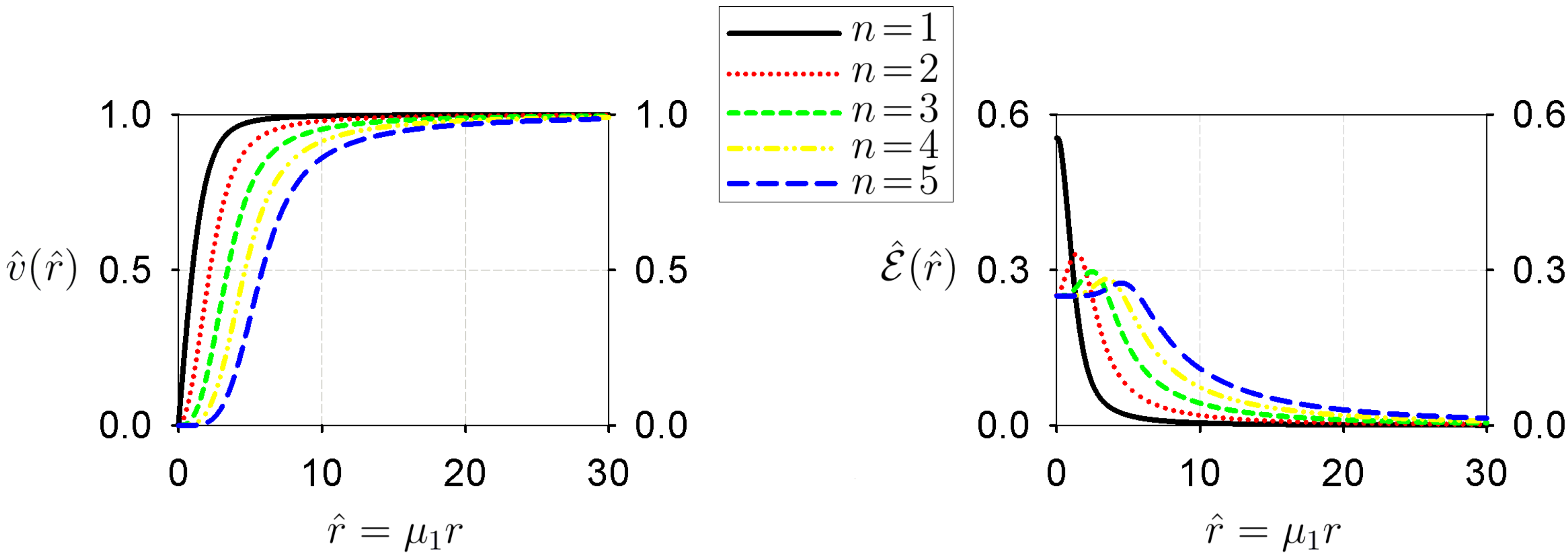


with

$$\mu^2 \;=\; \frac{\mu_1^2}{\mu_2^2} \; .$$

- **CP3 Vortices** [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

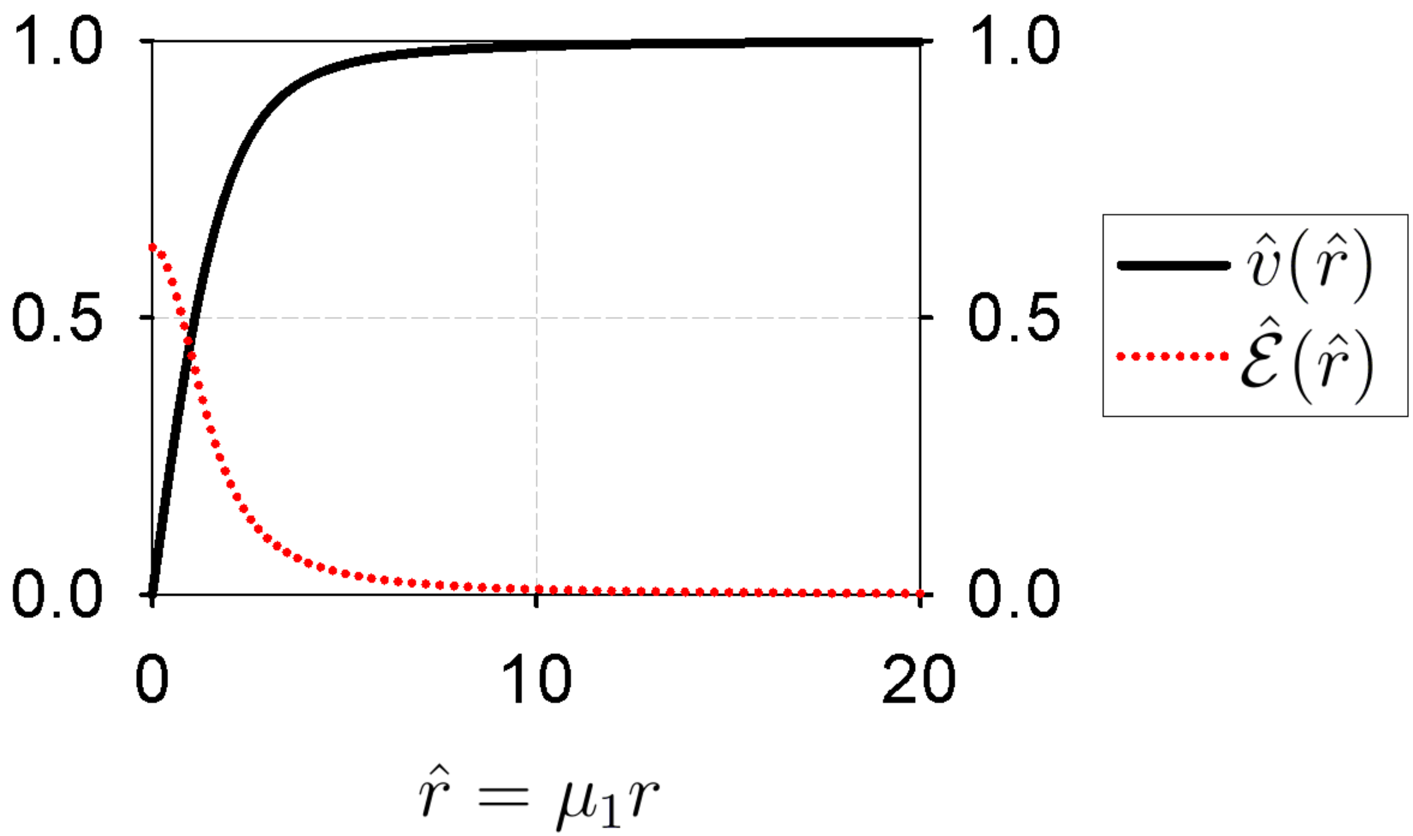
$$\phi_1(r,\chi)\ =\ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)\cos(n\chi) \end{pmatrix}\,,\qquad \phi_2(r,\chi)\ =\ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -v(r)\sin(n\chi) \end{pmatrix}\,.$$



• SO(3)$_{\mathrm{HF}}$ Global Monopole

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r,\chi) \;=\; \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v(r)\sin\chi \end{array}\right)\,, \qquad \phi_2(r,\chi,\psi) \;=\; \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v(r)e^{i\psi}\cos\chi \end{array}\right)\,.$$



• Conclusions

- **8D Majorana scalar-field formalism $\Longrightarrow$ SO(1,5) bilinear formalism.**

- **The 2HDM potential may exhibit up to <u>13</u> different discrete and global symmetries in the SO(1,5) bilinear formalism.**

- **The $U(1)_{\rm Y}$-violating 2HDM potential can possess up to <u>15</u> distinct symmetries in the SO(1,5) bilinear formalism.**

- **Topological defects resulting from the spontaneous breaking of 6 generic HF/CP symmetries in the 2HDM:**

Domain Walls:	Z_2; CP1; CP2,
Vortices:	$U(1)_{\rm PQ}$; CP3,
Monopoles:	$SO(3)_{\rm HF}$.

- **Study of the spatial profile of topological defects.**

• Future Directions

- Study of the breaking pattern of the additional 7 symmetries.
- Domain wall constraints on masses and couplings in the 2HDM.
- Cosmological implications of strings and global monopoles.
- Phenomenological implications of (un)stable topological defects for the LHC.
- Study of the 15 symmetries in the U(1)-Violating 2HDM.
- $\cdots$