Bulk Higgs in Warped Extra Dimensions
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The outline of this talk is

- Introduction
- General results
- RS model
- Our model
- Flavor violation (preliminary)
- Conclusion
INTRODUCTION

- The SM of EW interactions suffers from a naturalness problem as the Higgs mass is sensitive to UV cutoff $\Lambda$.
- This is the

Hierarchy (Naturalness) Problem

- A number of SM extensions have been proposed with the aim of solving the hierarchy problem.
- One of the most interesting solutions (warped models) were originally proposed by Randall and Sundrum (RS) $^1$ based on an extra dimension with AdS metric.

AdS metric

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad A(y) = ky$$

and UV and IR boundaries located at $y = 0$ and $y = y_1$.

The original model had all the SM localized on the IR boundary and the Planckian Higgs mass redshifted to the TeV scale by the warp factor.

Now the SM is not necessarily localized at the IR boundary.

- If fermions (and gauge bosons) propagate in the bulk with $O(k)$ Dirac masses the model could provide a theory of flavor.
- If the Higgs propagate in the bulk it can solve the hierarchy problem provided that it is "sufficiently localized" towards the IR boundary.

However, confronting the model with Electroweak Precision Tests (EWPT):

- Implies strong bounds on the KK modes:
  - $m_{KK} \gtrsim 13$ TeV (for localized Higgs)
  - No hope for LHC searches
  - Re-creating a little hierarchy problem
The origin of the previous bound is the large volume compactification and the behavior of the $T$ parameter

$$\alpha T \sim k y_1$$

Some solutions to this problem have been proposed
- To enlarge the 5D model to incorporate a gauge custodial (bulk) symmetry $^2$ (extension of 5D SM)
- To introduce a large IR brane kinetic terms $^3$ (relies on unknown UV physics)
- To generalize the AdS metric in the IR with a strong deformation of conformality such that the coupling of EW KK modes to the Higgs is suppressed $^4$

We will consider here the last solution
We will present results for general metrics, Higgs and fermion profiles

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$^3$ M. Carena et al., arXiv:hep-ph/0212307
$^4$ J. Cabrer et al., arXiv:1011.2205
**General results**

- We will then consider the SM propagating in a 5D space with an arbitrary metric $A(y)$
- 5D gauge fields $g_M(x,y)$, $W^i_M(x,y)$ and $B_M(x,y)$ propagating in the bulk
- A stabilizing field $\phi(x,y)$ fixing the value of $A(y_1)$

**A bulk SM Higgs**

$$H(x,y) = \frac{1}{\sqrt{2}} e^{i\chi(x,y)} \begin{pmatrix} 0 \\ h(y) + \xi(x,y) \end{pmatrix}$$

- $\chi(x,y)$ contains the 4D Goldstone bosons
- $h(y)$ is the 5D Higgs background
- $\xi(x,y)$ describes the Higgs fluctuations

- We will consider for the moment an arbitrary background $h(y)$
The effective SM-like Lagrangian for the Higgs

\[ \mathcal{L}_{\text{eff}} = -|D_\mu \mathcal{H}|^2 + \mu^2 |\mathcal{H}|^2 - \lambda |\mathcal{H}|^4 \]

\[ \mu^2 \sim Z^{-1} \rho^2, \quad \lambda \sim Z^{-2} \]

The IR scale \( \rho \) and dimensionless quantity \( Z \) are

\[ \rho = ke^{-A(y_1)}, \quad Z = k \int_0^{y_1} dy \frac{h^2(y)}{h^2(y_1)} e^{-2A(y)+2A(y_1)} \]

\( Z \) is Higgs wave function renormalization

The physical Higgs mass is \( m_H^2 = 2\mu^2 \sim 2Z^{-1}\rho^2 \)

Radiative corrections in the effective theory below the scale \( \Lambda \sim m_{KK} \) will tend to destabilize light Higgs masses: some degree of fine-tuning is needed to not spoil EWSB and to keep the Higgs light.
UNIVERSAL ELECTROWEAK OBSERVABLES

- Universal electroweak precision observables are commonly mapped to the set \((T, S, W, Y)\)
- They are defined as

**Oblique observables**

\[
\begin{align*}
\alpha T &= m_W^{-2} \left[ c_W^2 \Pi_Z(0) - \Pi_W(0) \right] \\
\alpha S &= 4 s_W^2 c_W^2 \left[ \Pi'_Z(0) - \Pi'_\gamma(0) \right] \\
2m_W^{-2} Y &= s_W^2 \Pi''_Z(0) + c_W^2 \Pi''_\gamma(0) \\
2m_W^{-2} W &= c_W^2 \Pi''_Z(0) + s_W^2 \Pi''_\gamma(0)
\end{align*}
\]

- Associated with the coefficients of

**Effective operators (d=6)**

\[|H^\dagger D_\mu H|^2, \quad H^\dagger W^\mu_\nu HB^\mu_\nu, \quad (\partial_\rho B^\mu_\nu)^2, \quad (D_\rho W^\mu_\nu)^2\]

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\(^5\) R. Barbieri et al. hep-ph/0405040
These observables can be computed as

**Oblique observables**

\[
\begin{align*}
\alpha T &= s_W^2 m_Z^2 \frac{l_2}{\rho^2} \frac{ky_1}{Z^2} \\
\alpha S &= 8s_W^2 c_W^2 m_Z^2 \frac{l_1}{\rho^2} \frac{1}{Z} \\
Y &= W = c_W^2 m_Z^2 \frac{l_0}{\rho^2} \frac{1}{ky_1}
\end{align*}
\]

where

\[
I_n = k^3 \int_0^{y_1} (y_1 - y)^{2-n} u^n(y) e^{2A(y) - 2A(y_1)}
\]

\[
I_n/\rho^2 = \mathcal{O}(1/m_{KK}^2)
\]

- \( T \) is volume enhanced and \( Z^2 \) suppressed
- \( S \) is \( Z \) suppressed and \( W = Y \) is volume suppressed
FERMION MASSES (PRELIMINARY)

Consider the fermion action $^6$

$$S = \int dy \, e^{-3A} \left( i \bar{\psi}_L \partial_y \psi_L + i \bar{\psi}_R \partial_y \psi_R \right)$$

$$+ e^{-4A} \left( \bar{\psi}_R \psi'_L - 2A' \bar{\psi}_R \psi_L - M(y) \bar{\psi}_R \psi_L + \text{h.c.} \right)$$

There is a zero mode with profile

$$\psi_{L,R} = e^{2A(y)-Q(y)_{L,R}}$$

where $Q(y)_{R,L} = \pm \int_0^y M(y')$

We can make the choice $Q_f(y) = c_f A(y)$ which coincides with that used in RS models where $Q_f^{RS} = c_f k y$

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$^6$ J. Cabrer, G. Gersdorff, M.Q., in preparation
A very useful quantity is

\[ \Omega_f(y) = \frac{U_f(y)}{U_f(y_1)}, \quad U_f(y) = \int_0^y \exp \left[ (1 - 2c_f)A(y') \right] \]

The 4D Yukawa couplings for quarks are given by

\[ Y^q_{ij} = \hat{Y}^q_{ij} \ F(c_{Q_i}^L, c_{q_j}^R), \quad q = u, d \]

\[ F(c_1, c_2) = \frac{\int h(y)e^{-(c_1+c_2)A(y)}}{\left[ \int h^2(y)e^{-2A} \int e^{(1-2c_1)A} \int e^{(1-2c_2)A} \right]^{1/2}} \]

Here \( \hat{Y} \) is the 5D Yukawa coupling with mass dimension \(-\frac{1}{2}\)

If flavor is explained by different localization in the extra dimension there is flavor violation mainly from KK gluons
**FLAVOR VIOLATION** *(PRELIMINARY)*

- The dominant flavor violation comes from the KK gluons
- We parametrize the most constraining

\[ \Delta F = 2 \text{ Lagrangian} \]

\[ H_{sd}^{\Delta F=2} = -C_{1}^{sd}(\bar{s}_L \gamma^\mu d_L)^2 - \tilde{C}_{1}^{sd}(\bar{s}_R \gamma^\mu d_R)^2 + C_{4}^{sd}(\bar{s}_L d_R)(\bar{s}_R d_L) \]

- With

**Wilson coefficients**

\[ C_{1}^{sd} = \frac{g_s^2 y_1}{6} \int e^{2A}(\Omega^{21}_{d_L})^2 \]

\[ \tilde{C}_{1}^{sd} = \frac{g_s^2 y_1}{6} \int e^{2A}(\Omega^{21}_{d_R})^2 \]

\[ C_{4}^{sd} = -g_s^2 y_1 \int e^{2A}\Omega^{21}_{d_L}\Omega^{21}_{d_R} \]
Where \(^7\)

\[\Omega^{ij}_{q\chi} = (V_{q\chi} \Omega^{diag}_{q\chi} V^\dagger_{q\chi})^{ij}\]
\[\chi = L, R \quad q = u, d\]

Using

Unitarity of transformations

\[\Omega^{21}_{d_L} = (\Omega^2_{d_L} - \Omega^1_{d_L}) V^{22}_{d_L} V^{*12}_{d_L} + (\Omega^3_{d_L} - \Omega^1_{d_L}) V^{23}_{d_L} V^{*13}_{d_L}\]

and similarly for \(L \rightarrow R\)

\(^7\) J. Cabrer, G. Gersdorff, M.Q., in preparation
RS Model

- In the RS model one introduces a 5D bulk Higgs mass: 
  \[ M^2 = a(a - 4)k^2 \]
- Holographic interpretation: \( a = \dim(O_H) \)
- Solution to the EOM: \( h(y) \propto e^{aky} \)
- No fine-tuning for \( a > 2 \): hierarchy solved with a composite Higgs
- \( Z = \frac{1}{2(a-1)} < 1/2 \) provides no suppression \( \Rightarrow \) large \( m_{KK} \) & heavy Higgs & 5D Higgs as IR delocalized as possible
- For instance for \( m_H = 150 \) GeV the 95% CL bound is

<table>
<thead>
<tr>
<th>For localized 5D Higgs: ( a \to \infty )</th>
<th>( m_{KK} \gtrsim 12.5 ) TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>For delocalized 5D Higgs: ( a = 2.1 )</td>
<td>( m_{KK} \gtrsim 7 ) TeV</td>
</tr>
</tbody>
</table>
95% CL regions in the \((m_H, m_{KK})\) plane for RS and the cases of a localized and a bulk Higgs with \(a = 2.1\). Solid line is the perturbativity bound \(\beta^{(2)} = 0.5 \beta^{(1)}\)
**Our model**

- We will consider a model with a *conformal deformation* in the IR
- It contains a *stabilizing field* $\phi$ which leads to

**The metric**

$$A(y) = ky - \frac{1}{\nu^2} \log \left(1 - \frac{y}{(y_1 + \Delta)}\right), \quad \nu \in \mathbb{R}$$

- The metric has a *spurious singularity* located at $y_s = y_1 + \Delta$ outside the physical interval
- The dynamics of $\phi$ fixes $y_1 [A(y_1)]$ and $\Delta$ as in GW
- A 5D bulk Higgs mass:
  $$M^2(\phi) = k^2 [a(a - 4) - be^{\nu \phi}]$$
  where $a$ and $b$ are arbitrary constants
- $b$ can be absorbed by a shift of $\phi_0$ at the UV: we fix it to $b = 1$
The holographic interpretation of $a$ is now a bit different from RS.

In the IR

\[
\text{dim}(\mathcal{O}_H)^{IR} = \frac{a}{1 + \frac{1}{k\Delta\nu^2}}
\]

The solution to the EOM

\[h(y) = c_1 e^{aky} + c_2 \int^y e^{4A(y')-2aky'}\]

imposes the constraint

Hierarchy condition ($a > 2$ for RS)

\[a \gtrsim a_0 = 2A_1/ky_1\]

In many cases $Z \gg 1$ which softens the bounds on EWPO.
Contour lines of fixed $Z(\nu, \Delta)$ where $a = a_0(\nu, \Delta)$ and $A(y_1) = 35$
95% CL regions. Ray (c) [(d)] is our model with $k\Delta = 1$ and $\nu = 0.7$ [$\nu = 0.6$]
For instance for $m_H = 150$ GeV and $a = a_0$ the 95% CL bound is

For $k\Delta = 1, \nu = 0.7$

$$m_{KK} \gtrsim 3 \text{ TeV}$$

For $k\Delta = 1, \nu = 0.6$

$$m_{KK} \gtrsim 2 \text{ TeV}$$

This shows that:

- A light or heavy Higgs can be consistent with KK-modes accessible at LHC energies
- The measurement of the Higgs mass at LHC should constrain the model parameters
- These two features are exhibited in the next plot
95% CL regions in the \((m_H, m_{KK})\) plane for our model with \(k\Delta = 1\) and \(\nu = 0.7, 0.6, 0.5\). Solid line corresponds to the perturbativity bound \(\beta_\lambda^{(2)} = 0.5 \beta_\lambda^{(1)}\).
The nine 5D quark masses $c_{Q^i_L}$, $c_{u^i_R}$, $c_{d^i_R}$ are adjusted to satisfy the quark mass and CKM matrix relations.

- Dominant flavor violation comes from the KK gluons.
- The general trend on the coupling of KK gluons to fermions in our model as compared to the RS model.
To quantify the improvement that we can achieve within our model, we have randomly generated complex 5D Yukawa couplings and performed a $\chi^2$ fit to the nine parameters $c_f$

We have considered both RS and our model with $\nu = 0.5, \; \Delta = 1/k$

Each set of 5D Yukawas thus gives rise to two sets of $c_f$, one for RS and one for our model.

The $c_f$ for our model are consistently larger than in RS, giving rise to reduced and more universal couplings.

With these data points we then compute the exact mixing matrices numerically and use them to find the coefficients $C_i$: in particular

$$|\text{Im } C_{4}^{sd}| \lesssim 2.6 \times 10^{-17} \text{ GeV}^{-2} \; 95\% \; C.L.$$

which is the most constrained one from data.

These are then translated into bounds on $m_{KK}$
We have considered 2,000 randomly chosen 5D Yukawas with $1 < \sqrt{k}|\hat{Y}_{ij}| < 2$

**Figure:** Cumulative distribution function of the bounds in the cases of RS (red line) and our model (blue line). We also show the median of the two distributions (dashed lines)
For the same distribution ${}^8$

![Graph: Cumulative distribution function of the bounds ratio $m_{\nu=0.5}^{KK}/m_{KK}^{RS}$]

**Figure:** Cumulative distribution function of the bounds ratio $m_{\nu=0.5}^{KK}/m_{KK}^{RS}$

- We obtain a probability (percentile) 80% (50% median) for

$$m_{\nu=0.5}^{KK} < m_{KK}^{RS} (m_{KK}^{\nu=0.5} < 0.4m_{KK}^{RS})$$

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8 J. Cabrer, G. Gersдорff, M.Q., in preparation
CONCLUSIONS

- We have considered models where the 5D SM gauge and Higgs bosons propagate in the bulk
- In the RS model a heavy Higgs is more natural than a light one for which there is a ”little hierarchy problem”

RS (non-custodial) bound for $m_H = 115$ GeV

$$(S, T, W, Y) \Rightarrow m_{KK} \gtrsim 7 \text{ TeV} \ @ \ 95\% \ CL$$

- In the model with an IR deformation the KK spectrum can be accessible to LHC both for light and heavy Higgs

Our model with $k\Delta = 1$ and $\nu = 0.6$ for $m_H = 115$ GeV

$$(S, T, W, Y) \Rightarrow m_{KK} \gtrsim 2 \text{ TeV} \ @ \ 95\% \ CL$$

- Higgs discovery at LHC will constrain the parameters!
If fermions are conveniently localized toward the fifth dimension a

Theory of Flavor

can be constructed

- The \( c_f \) for our model are consistently larger than in RS, giving rise to reduced and more universal couplings

Flavor Violation

- Flavor Violation appears dominantly from KK gluons
- A random choice of 5D Yukawas also leads to improvement in our model with respect to RS
- We are at present implementing other observables \( Zb\bar{b}, C_1, \ldots \) in our model \(^9\)

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