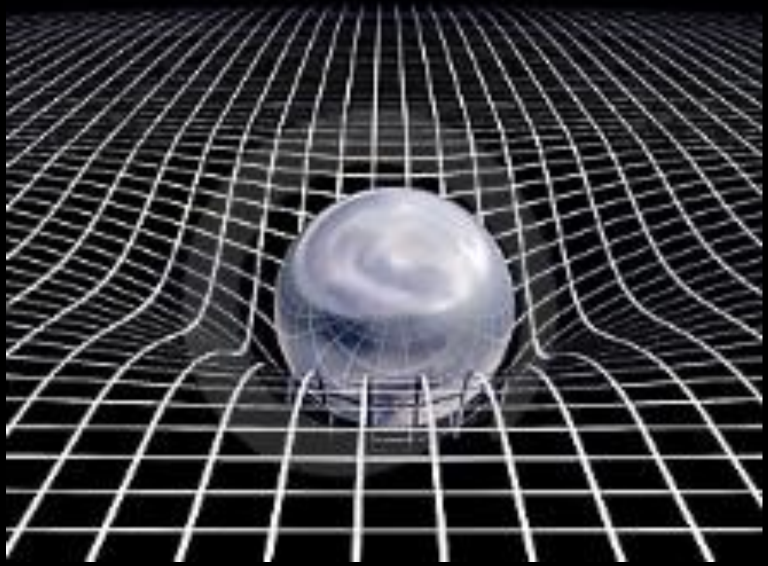

Good Things From Brane Back-reaction

Natural hierarchies and cosmology from higher codimension branes

w Leo van Nierop

Light Scalars from Low-Scale Gravity

Natural hierarchies and cosmology from higher codimension branes

Scalars 2011

w Leo van Nierop

Outline

- Motivation
  - Naturalness and light scalars

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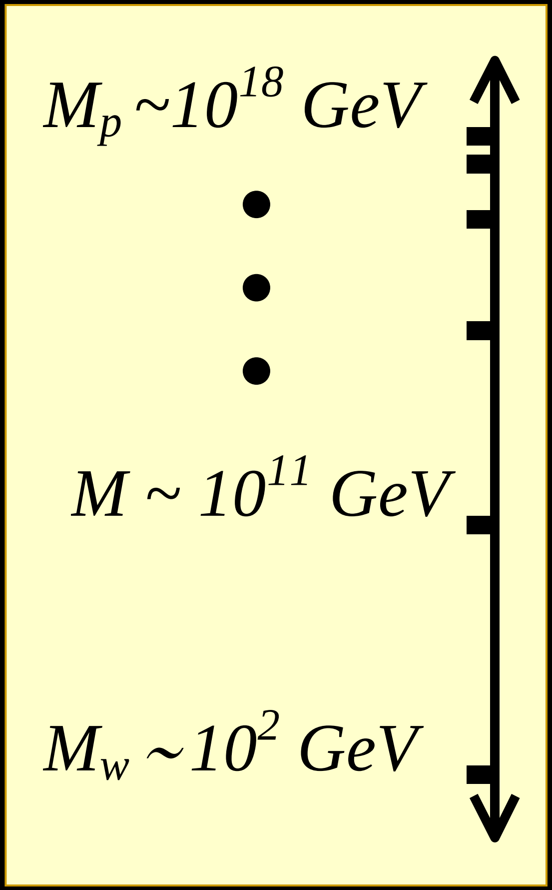
  - Naturalness and light scalars
- Light scalars from extra dimensions
  - The generic situation
  - Brane back-reaction as a game-changer

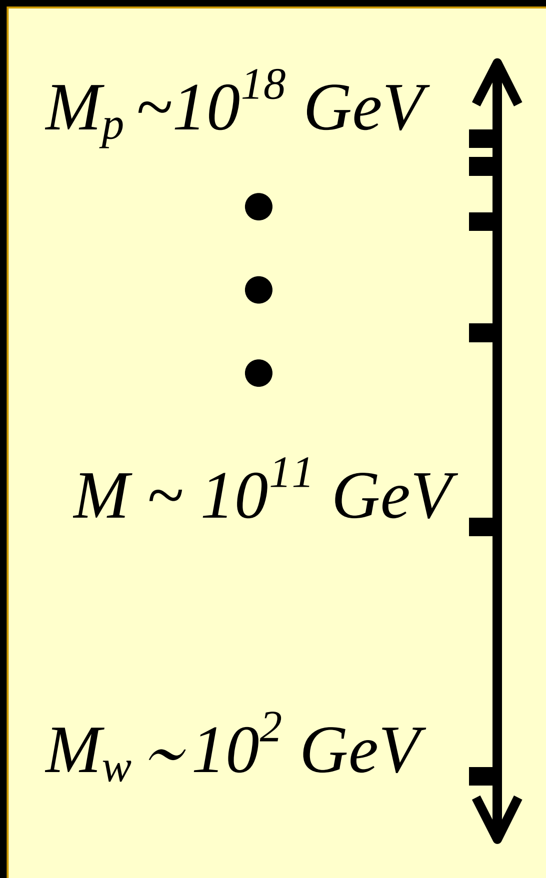
Naturalness



effective theory can be defined at many scales

$$m^2 \approx m_0^2 + \cdots$$

Naturalness

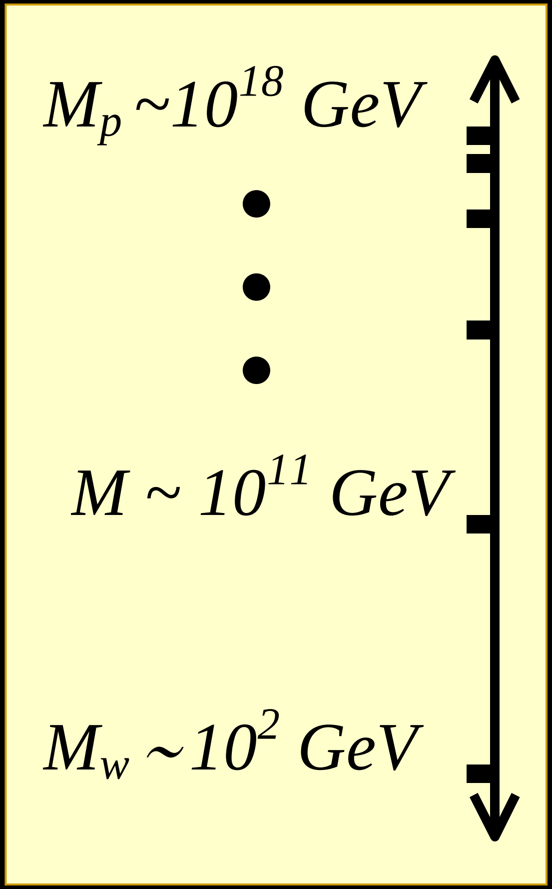


effective theory can be defined at many scales

$$m^2 \approx m_1^2 + kM^2 + \cdots$$

$$m^2 \approx m_0^2 + \cdots$$

Naturalness



effective theory can be defined at many scales

$$m^2 \approx m_1^2 + k M^2 + \cdots$$

$$m^2 \approx m_0^2$$

Must cancel to 20 decimal places!!

Naturalness in Crisis

$m_w \sim 10^{11}\ eV$

$\vdots$

$m_\mu \sim 10^{8}\ eV$

$m_e \sim 10^{6}\ eV$

$m_\nu \sim 10^{-2}\ eV$

Can apply same argument to scales between TeV and sub-eV scales.

$$\mu^4 \approx \underbrace{\mu_1^4 + k_e m_e^4}_{\mu_0^4} + k_\nu m_\nu^4$$

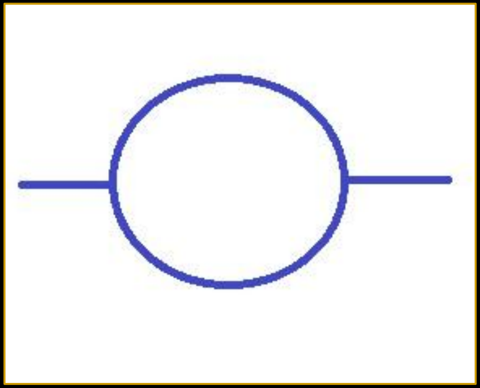
$$\mu^4 \approx \mu_0^4 + k_\nu m_\nu^4$$

Must cancel to 32 decimal places!!

Scalars and Naturalness

- Scalar masses get large corrections even if the scalars couple only with gravitational strength:

$$\delta m^2 = \frac{\Lambda^4}{{M_4}^2} = {M_4}^2 \quad \text{if } \Lambda = M_4$$



Scalars and Naturalness

- Party line: for energies too low for SUSY to help, a pseudo-Goldstone boson is the only option for naturally light scalars

$$L = f^2(\partial\vartheta)^2 + \mu^4 U(\vartheta) + \cdots$$

$$m^2 = \frac{\mu^4}{f^2} \quad \text{is protected by} \quad \vartheta \to \vartheta + c$$

Outline

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  - The generic situation
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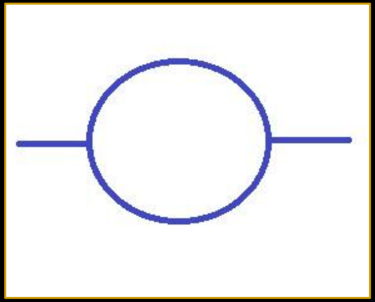
Branes and Naturalness

- One way extra dimensions help is by lowering the gravity scale: e.g. $M_4 = {M_6}^2 r$ in 6D

$$\delta m^2 = \frac{\Lambda^4}{{M_4}^2} = \frac{{M_6}^4}{{M_4}^2} = \frac{1}{r^2} \quad \text{if } \Lambda \leq M_6$$



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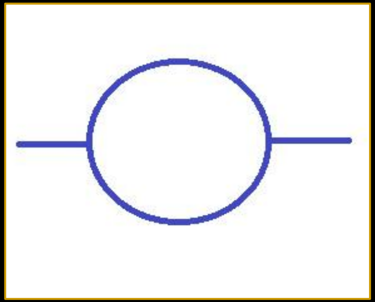
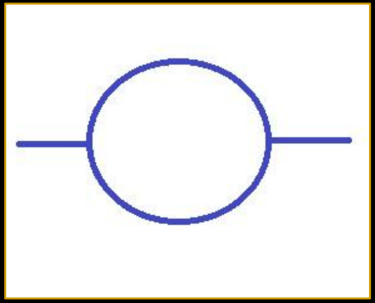


Seems cannot get *m* smaller than smallest KK scale: 10^{-2} eV

Branes and Naturalness

- One way extra dimensions help is by lowering the gravity scale: e.g. $M_4 = {M_6}^2 r$ in 6D

$$\delta m^2 = \frac{\Lambda^4}{{M_4}^2} \leq \frac{1}{r^4 {M_4}^2} \ \text{ if } \ \Lambda \leq \frac{1}{r}$$

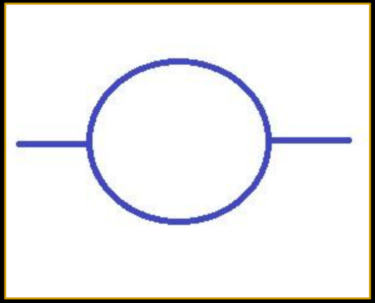


But 4D kinematics cannot be used for the loop for Λ larger than the KK scale

Branes and Naturalness

- Must re-estimate the contributions from higher dimensions using higher dimensional kinematics

$$\delta m^2 = \frac{\Lambda^6}{{M_6}^4} = {M_6}^2 \ \text{ if } \ \Lambda = M_6$$



Same as 4D estimate if one sums over $N \sim (M_6\, r)^2$ KK states

Branes and Naturalness

- *But* a scalar in 4D need not be a scalar in higher dimensions, and so its mass can be protected from higher-dimensional loops

Rest of talk: In some circumstances, can get masses as low as $\delta m \sim 1/(M_4 r^2)$, *which can be as low as the Hubble scale if r is as large as possible*

Branes and Naturalness

- Suppose scalar is a modulus of the metric, like *r* itself. UV loop gives local contribution to action

$$
\begin{aligned}
\mathrm{V} &= \int d^2x \left(c_0 M^6 + c_1 M^4 R + c_2 M^2 R^2 + c_3 R^3 + \right) \\
&= r^2 \left(c_0 M^6 + c_1 \frac{M^4}{r^2} + c_2 \frac{M^2}{r^4} + c_3 \frac{1}{r^6} + \cdots \right)
\end{aligned}
$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = {M_4}^2(\partial r/r)^2$ the resulting mass contributions are $\delta m^2 = V/{M_4}^2$

$$\mathrm{V} = r^2\left(c_0 M^6 + c_1 \frac{M^4}{r^2} + c_2 \frac{M^2}{r^4} + c_3 \frac{1}{r^6} + \cdots\right)$$

$$m^2 = \left(c_0 \frac{M^6}{{M_6}^4} + c_1 \frac{M^4}{{M_4}^2} + c_2 \frac{M^2}{{M_6}^4 r^4} + c_3 \frac{1}{{M_6}^4 r^6} + \right)$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = M_4{}^2(\partial r/r)^2$ the resulting mass contri... $M_4{}^2$

$$V = r^2\left(c_0 M^6 + c_1 \ldots \ldots\right)$$

This gives the naïve 4D result:

$$V = N\,M^4 = r^2\,M^6$$

$$m^2 = \left(c_0\frac{M^6}{M_6{}^4} + c_1\frac{M^4}{M_4{}^2} + c_2\frac{M^2}{M_6{}^4 r^4} + c_3\frac{1}{M_6{}^4 r^6} + \right)$$

Branes and Naturalness

- Given the kinetic term L_{ki}… resulting mass contributio…

This gives the naïve KK contribution

$m = M^2/M_4 = 1/r$

$$\mathrm{V} = r^2\left(c_0 M^6 + c_1 \frac{M^4}{r^2} + \cdots\right)$$

$$m^2 = \left(c_0 \frac{M^6}{{M_6}^4} + c_1 \frac{M^4}{{M_4}^2} + c_2 \frac{M^2}{{M_6}^4 r^4} + c_3 \frac{1}{{M_6}^4 r^6} + \right)$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = M_4{}^2(\partial r/r)^2$ the resulting $m^2 = V/M_4{}^2$

This is gives $V \sim 1/r^4$ so could be the right size to be Dark Energy

$m = 1/(M_4\, r^2) = H$

$$V = r^2 \left(\cdots + c_3 \frac{1}{r^6} + \cdots \right)$$

$$m^2 = \left(c_0 \frac{M^6}{M_6{}^4} + c_1 \frac{M^4}{M_4{}^2} + c_2 \frac{M^2}{M_6{}^4 r^4} + c_3 \frac{1}{M_6{}^4 r^6} + \right)$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = M_4{}^2(\partial r/r)^2$ the resulting mass contri… $_4{}^2$

$$V = r^2\left(c_0 M^6 + c_1 \cdots \right)$$

Do systems exist for which first 3 terms vanish?

$$m^2 = \left(c_0\frac{M^6}{M_6{}^4} + c_1\frac{M^4}{M_4{}^2} + c_2\frac{M^2}{M_6{}^4 r^4} + c_3\frac{1}{M_6{}^4 r^6} + \right)$$

Branes and Naturalness

- Given … he
  resulti … $_4{}^2$

YES!

Back-reaction can cancel the rest for two supersymmetric extra dimensions

$$V = r \cdots \cdots \Big)$$

$$m^2 = \left(c_0 \frac{M^6}{{M_6}^4} + c_1 \frac{M^4}{{M_4}^2} + c_2 \frac{M^2}{{M_6}^4 r^4} + c_3 \frac{1}{{M_6}^4 r^6} + \right)$$

Setup

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}[R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

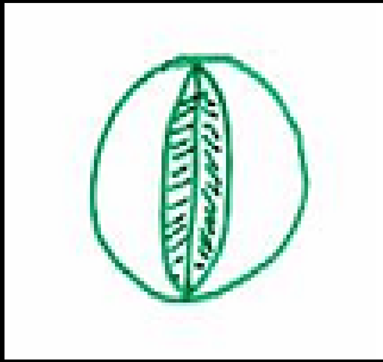
- Two specific cases
  - 6D axion: $a = 0$ and $V = \Lambda$
  - 6D supergravity: $a = 1$ and $V = \frac{2g_R^2}{\kappa^4} e^{\phi}$

Setup

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-a\phi_0} \qquad \phi = \phi_0$$



Setup

- Simple solution (including back-reaction)

$$ds^2 = \hat{g}_{mn} dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

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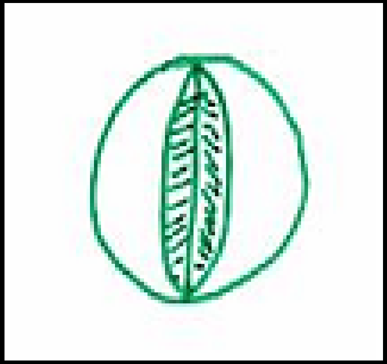

$$1 - \alpha = \frac{\kappa^2 T}{2\pi}$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2$$

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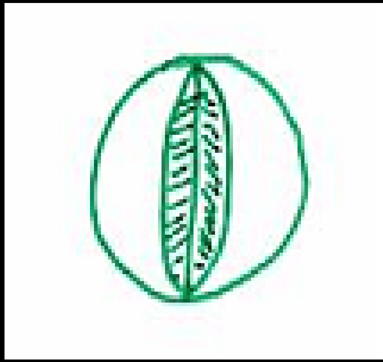

Field equations

$$\frac{2}{L^2} = \kappa^2\left(\frac{3Q^2}{2} + \Lambda\right)$$

$$\hat{R} = \kappa^2(Q^2 - 2\Lambda)$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2$$

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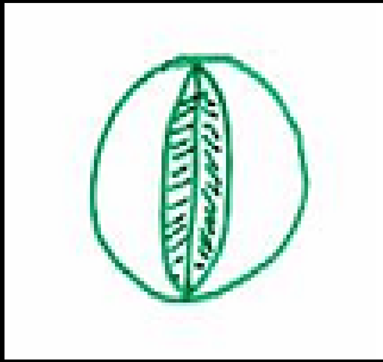

$$Q = \frac{n}{2\alpha g L^2} \qquad\qquad \hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

$$\frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2\kappa^2}\left[1 \mp \sqrt{1 - \left(\frac{3n^2\kappa^4\Lambda}{8\alpha^2 g^2}\right)}\right]$$

Setup

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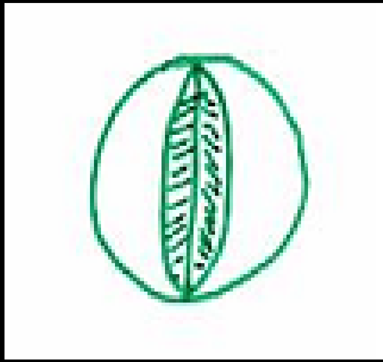


Tune $\Lambda = \frac{Q^2}{2}$ so $\hat{R} = 0$

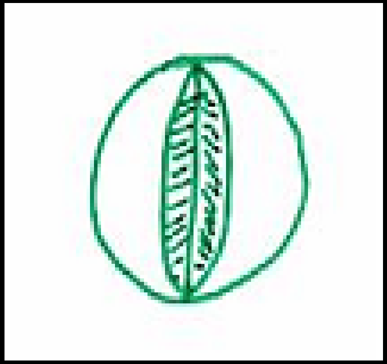
If $T \to T + \delta T$ then $\hat{R} \to -\frac{\kappa^2 \rho}{\pi \alpha L^2}$ where $\rho = 2\,\delta T$

Setup

- Simple solution (SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-\phi_0} \qquad \phi = \phi_0$$



Field equations

$$\frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2}$$

$$\kappa^2 Q^2 L^2 = 1 \qquad \hat{R} = 0$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$ $\boxed{\text{Obstructs } T \text{ to } \delta T}$

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Obstructs T to δT

- On other hand, general argument:

$$\rho = \int dV\, L_{bulk} = -\frac{1}{2\kappa^2}\int dV\ \partial^2\phi = \oint dS\, n\cdot\partial\phi \ \propto\ \frac{\partial T}{\partial \phi}$$

Setup

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi)\ {}^*F + \ldots$$

Setup

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi)\ {}^*F + \ldots$$

- New function Φ has interpretation as brane-localized flux

$$\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b \, e^{\phi}$$

Calculation

- More general solutions

$$ds^2 = e^{2W}\hat{g}_{mn}dx^m\, dx^n + dr^2 + e^{2B}d\theta^2$$

$$F_{r\theta} = Qe^{B-4W} \qquad\qquad \phi = \phi(r)$$

Calculation

- Perturb brane properties

$$T \to T + \delta T(\phi)$$

- To evade time-dependence add current

$$\Delta L_{bulk} = J\phi \qquad \text{or} \qquad \Delta L_{bulk} = J$$

- Find general solution to linearized equations

$$\kappa^2 J L^2 \ll 1$$

Calculation

- Sample solutions

$$\delta W = W_0 + W_1 \cos\left(\frac{r}{L}\right)$$

$$\delta\phi = \phi_0 + \phi_1 \ln\left(\frac{1-\cos(r/L)}{\sin(r/L)}\right) - \kappa^2 J L^2 \ln\left[\sin\left(\frac{r}{L}\right)\right]$$

and so on

Calculation

- Brane-bulk boundary conditions:

$$(e^B \phi')_b = \frac{\kappa^2}{2\pi}\left(\frac{\partial L_b}{\partial \phi}\right)$$

$$(e^B W')_b = \frac{\kappa^2}{4\pi}\left(\frac{\partial L_b}{\partial g_{\theta\theta}}\right) = U_b$$

$$(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi}\left[\left(\frac{\partial L_b}{\partial \phi} + \frac{3}{2}\frac{\partial L_b}{\partial g_{\theta\theta}}\right)\right]$$

Constraint: $4U_b[2 - 2L_b - 3U_b] - \left(\frac{\partial L_b}{\partial \phi}\right)^2 = 0$

Calculation

- Non-SUSY result:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2}\left[\frac{\pi\alpha L^2 \hat{R}(\phi)}{\kappa^2}\right]$$

$$\left[\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

$$\rho = \left[\sum_b \delta T_b - 2Q\delta\Phi_b\right]_{\phi_*}$$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

ie Einstein frame potential: $V = U(\phi)e^{2\phi}$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

$$\rho = [\delta T_b - 2Q\delta\Phi_b] = \left[-\frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*}$$

Applications

- Three intriguing choices:

Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = Ce^{-\phi}$ then $V(\phi) = Ae^{2\phi}$

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Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = Ce^{-\phi}$ then $V(\phi) = Ae^{2\phi}$

Case 2: exponentially large volume:

$\delta T_b = A + B\,(\phi + v)^2$ with $v \sim 50$ then $r = Le^{-\phi/2} \gg L$

Applications

- Three intriguing choices:

Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

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Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

Brane action independent of ϕ stable against brane loops
Bulk loops generate corrections of order $e^{2\phi} = (1/r)^4$

Conclusions

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- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension one
- For codimension two:
  - Explicit matching between source and bulk known
  - Potentially useful applications: *very light scalars; exponentially large dimensions; progress on cosmological constant; de Sitter constructions…*

Fin