

Good Things From Brane Back-reaction

*Natural hierarchies and
cosmology from higher
codimension branes*



w Leo van Nierop



Light Scalars from Low-Scale Gravity

*Natural hierarchies and
cosmology from higher
codimension branes*

Scalars 2011

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PI

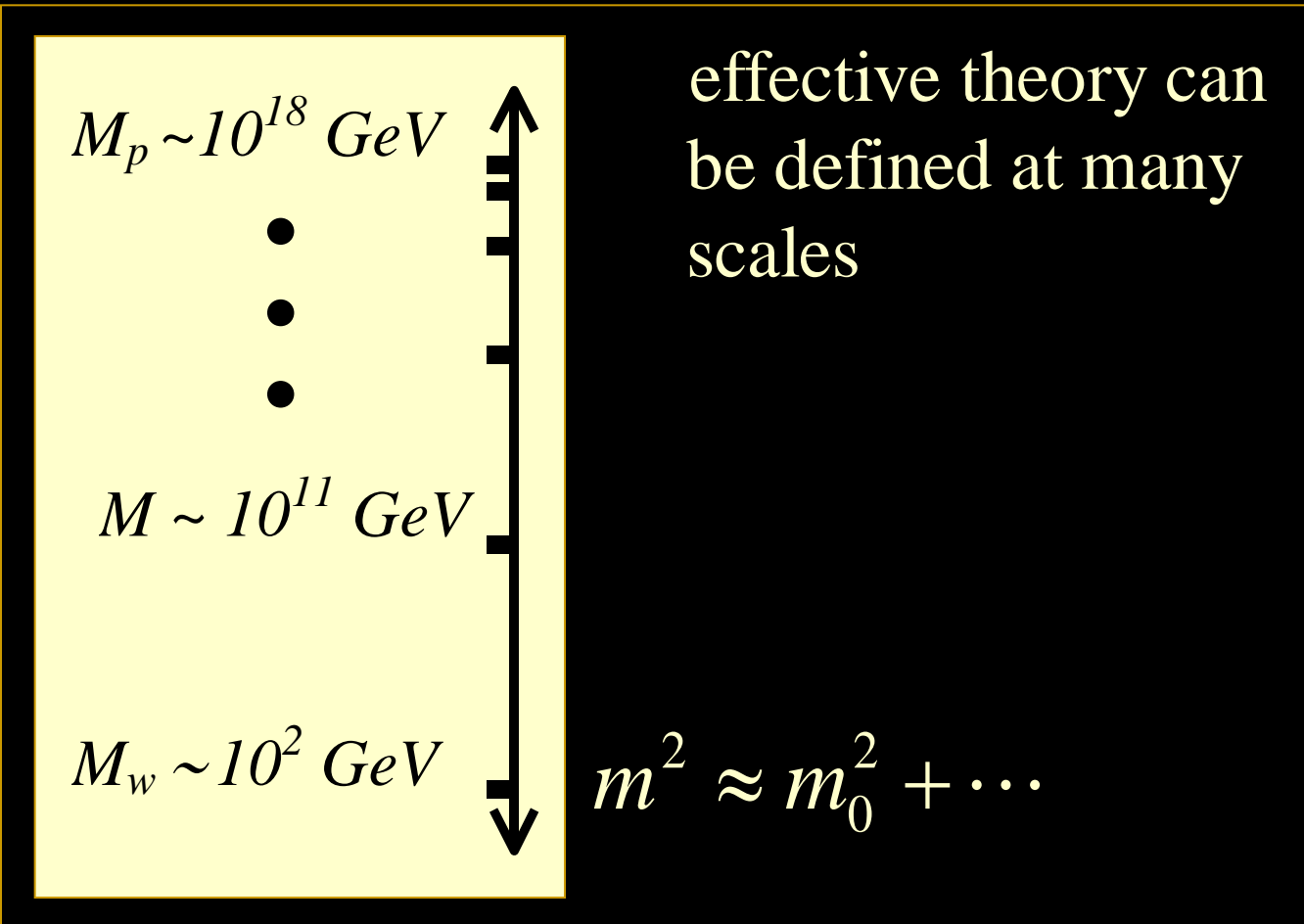
Outline

- Motivation
 - Naturalness and light scalars

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- Light scalars from extra dimensions
 - The generic situation
 - Brane back-reaction as a game-changer

Naturalness



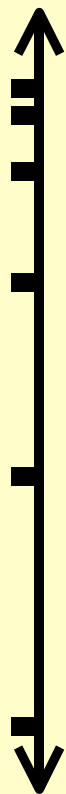
Naturalness

$$M_p \sim 10^{18} \text{ GeV}$$



$$M \sim 10^{11} \text{ GeV}$$

$$M_w \sim 10^2 \text{ GeV}$$



effective theory can
be defined at many
scales

$$m^2 \approx m_1^2 + kM^2 + \dots$$


$$m^2 \approx m_0^2 + \dots$$

Naturalness

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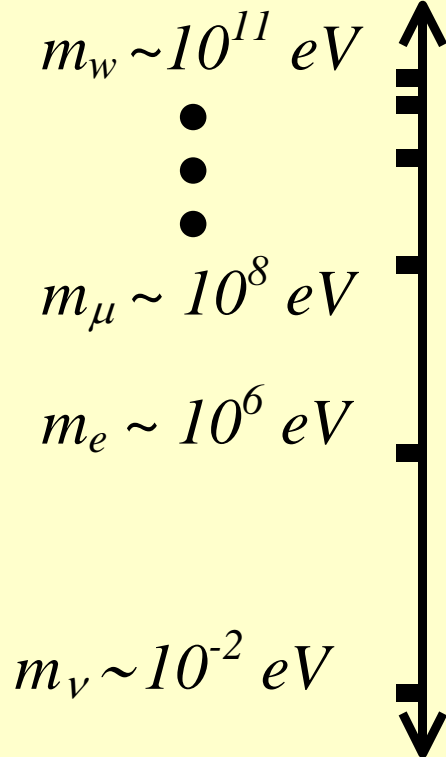
effective theory can be defined at many scales

$$m^2 \approx m_1^2 + kM^2 + \dots$$

$m^2 \approx m_0^2$

Must cancel to 20 decimal places!!

Naturalness in Crisis



Can apply same argument to scales between TeV and sub-eV scales.

$$\mu^4 \approx \underbrace{\mu_1^4}_{\leftarrow} + k_e m_e^4 + k_\nu m_\nu^4$$
$$\mu^4 \approx \underbrace{\mu_0^4}_{\leftarrow} + k_\nu m_\nu^4$$

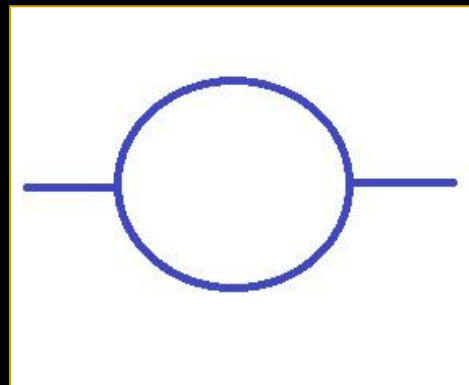
Arrows indicate that the $k_e m_e^4$ term in the first equation is being compared to the $k_\nu m_\nu^4$ term in the second equation.

Must cancel to 32 decimal places!!

Scalars and Naturalness

- Scalar masses get large corrections even if the scalars couple only with gravitational strength:

$$\delta m^2 = \frac{\Lambda^4}{M_4^2} = M_4^2 \quad \text{if } \Lambda = M_4$$



Scalars and Naturalness

- Party line: for energies too low for SUSY to help, a pseudo-Goldstone boson is the only option for naturally light scalars

$$L = f^2 (\partial\vartheta)^2 + \mu^4 U(\vartheta) + \dots$$

$$m^2 = \frac{\mu^4}{f^2} \text{ is protected by } \vartheta \rightarrow \vartheta + c$$

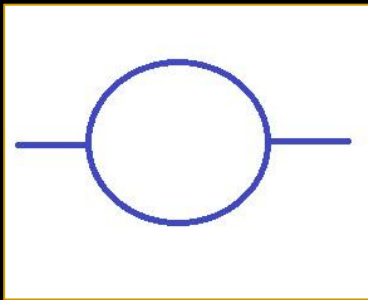
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Branes and Naturalness

- One way extra dimensions help is by lowering the gravity scale: e.g. $M_4 = M_6^2 r$ in 6D

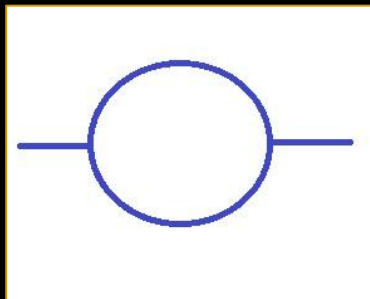
$$\delta m^2 = \frac{\Lambda^4}{M_4^2} = \frac{M_6^4}{M_4^2} = \frac{1}{r^2} \quad \text{if } \Lambda \leq M_6$$



Branes and Naturalness

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$$\delta m^2 = \frac{\Lambda^4}{M_4^2} = \frac{M_6^4}{M_4^2} = \frac{1}{r^2} \quad \text{if } \Lambda \leq M_6$$

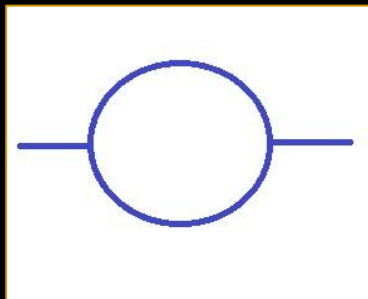


Seems cannot get m smaller than smallest KK scale: 10^{-2} eV

Branes and Naturalness

- One way extra dimensions help is by lowering the gravity scale: e.g. $M_4 = M_6^2 r$ in 6D

$$\delta m^2 = \frac{\Lambda^4}{M_4^2} \leq \frac{1}{r^4 M_4^2} \quad \text{if } \Lambda \leq \frac{1}{r}$$

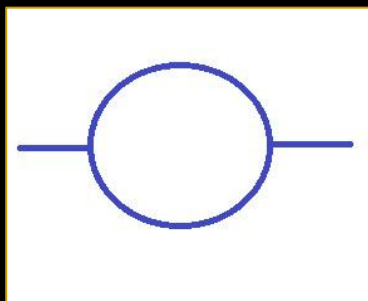


But 4D kinematics cannot be used for the loop for Λ larger than the KK scale

Branes and Naturalness

- Must re-estimate the contributions from higher dimensions using higher dimensional kinematics

$$\delta m^2 = \frac{\Lambda^6}{M_6^4} = M_6^2 \quad \text{if } \Lambda = M_6$$



Same as 4D estimate if one sums over $N \sim (M_6 r)^2$ KK states

Branes and Naturalness

- *But* a scalar in 4D need not be a scalar in higher dimensions, and so its mass can be protected from higher-dimensional loops

Rest of talk:

In some circumstances, can get masses as low as $\delta m \sim 1/(M_4 r^2)$, which can be as low as the Hubble scale if r is as large as possible

Branes and Naturalness

- Suppose scalar is a modulus of the metric, like r itself. UV loop gives local contribution to action

$$\begin{aligned} V &= \int d^2x (c_0 M^6 + c_1 M^4 R + c_2 M^2 R^2 + c_3 R^3 + \dots) \\ &= r^2 \left(c_0 M^6 + c_1 \frac{M^4}{r^2} + c_2 \frac{M^2}{r^4} + c_3 \frac{1}{r^6} + \dots \right) \end{aligned}$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = M_4^2 (\partial r / r)^2$ the resulting mass contributions are $\delta m^2 = V / M_4^2$

$$V = r^2 \left(c_0 M^6 + c_1 \frac{M^4}{r^2} + c_2 \frac{M^2}{r^4} + c_3 \frac{1}{r^6} + \dots \right)$$

$$m^2 = \left(c_0 \frac{M^6}{M_6^4} + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Branes and Naturalness

- Given the kinetic term $L_{\text{kin}} = M_4^2 (\partial r / r)^2$ the resulting mass contribution is $\propto M_4^2 r^2$

This gives the naïve 4D result:

$$V = N M^4 = r^2 M^6 \dots$$

$$V = r^2 \left(c_0 M^6 + c_1 \frac{M^4}{r^2} + c_2 \frac{M^2}{r^4} + c_3 \frac{1}{r^6} + \dots \right)$$

$$m^2 = \left(c_0 \frac{M^6}{M_6^4} + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Branes and Naturalness

- Given the kinetic term L_{kin} resulting mass contribution

This gives the naive
KK contribution

$$m = M^2/M_4 = 1/r$$

$$V = r^2 \left(c_0 M^6 + c_1 \frac{M^4}{r^2} + \dots \right)$$

$$m^2 = \left(c_0 \frac{M^6}{M_6^4} + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Branes and Naturalness

- Given the kinetic term $L_{kin} = M_4^2 (\partial r / r)^2$ the resulting

This is gives $V \sim 1/r^4$
so could be the right
size to be Dark Energy

$$m^2 = V/M_4^2$$

$$V = r^2 \left(c_0 \frac{M_6^6}{M_4^4} + c_1 \frac{M_4^4}{M_6^4} + c_2 \frac{M_6^2}{M_4^4 r^4} + c_3 \frac{1}{r^6} + \dots \right)$$

$$m = 1/(M_4 r^2) = H$$

$$m^2 = \left(c_0 \frac{M_6^6}{M_6^4} + c_1 \frac{M_4^4}{M_4^2} + c_2 \frac{M_6^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Branes and Naturalness

- Given the kinetic term $L_{\text{kin}} = M_4^2 (\partial r / r)^2$ the resulting mass contribution is

$$V = r^2 \left(c_0 M^6 + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + \dots \right)$$

Do systems exist for which first 3 terms vanish?

$$m^2 = \left(c_0 \frac{M^6}{M_6^4} + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Branes and Naturalness

- Given result

$$V = r$$

YES!

Back-reaction can cancel the rest for two supersymmetric extra dimensions

$$m^2 = \left(c_0 \frac{M^6}{M_6^4} + c_1 \frac{M^4}{M_4^2} + c_2 \frac{M^2}{M_6^4 r^4} + c_3 \frac{1}{M_6^4 r^6} + \dots \right)$$

Setup

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2} [R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

- Two specific cases

- 6D axion: $a = 0$ and $V = \Lambda$

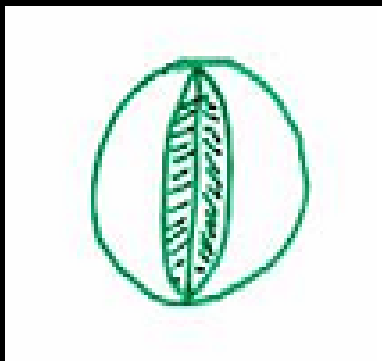
- 6D supergravity: $a = 1$ and $V = \frac{2g_R^2}{\kappa^4} e^\phi$

Setup

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L} \right) d\theta^2] e^{-a\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin \left(\frac{r}{L} \right) e^{-a\phi_0} \quad \phi = \phi_0$$

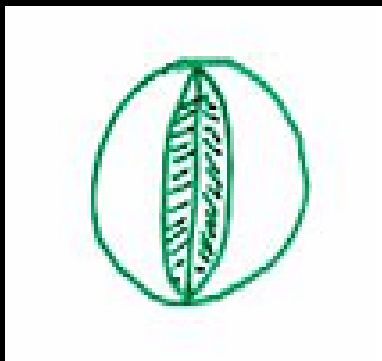


Setup

- Simple solution (including back-reaction)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-\alpha\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-\alpha\phi_0} \quad \phi = \phi_0$$



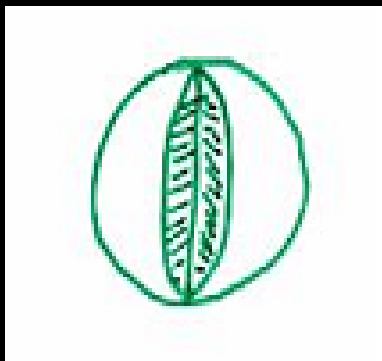
$$1 - \alpha = \frac{\kappa^2 T}{2\pi}$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) \quad \phi = \phi_0$$



Field equations

$$\frac{2}{L^2} = \kappa^2 \left(\frac{3Q^2}{2} + \Lambda \right)$$

$$\hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

Flux quantization

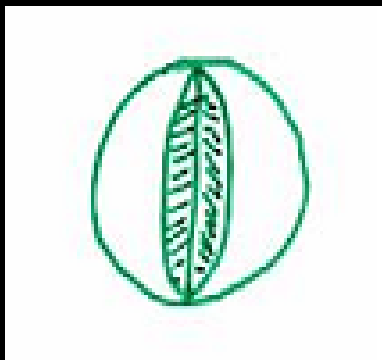
$$\frac{n}{g} = 2\alpha L^2 Q$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L} \right) d\theta^2$$

$$F_{r\theta} = Q\alpha L \sin \left(\frac{r}{L} \right) \quad \phi = \phi_0$$



$$Q = \frac{n}{2\alpha g L^2} \quad \hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

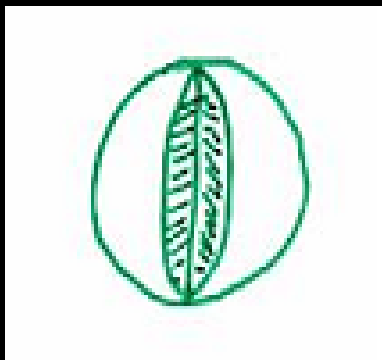
$$\frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2 \kappa^2} \left[1 \mp \sqrt{1 - \left(\frac{3n^2 \kappa^4 \Lambda}{8\alpha^2 g^2} \right)} \right]$$

Setup

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) \quad \phi = \phi_0$$



$$\text{Tune } \Lambda = \frac{Q^2}{2} \quad \text{so } \hat{R} = 0$$

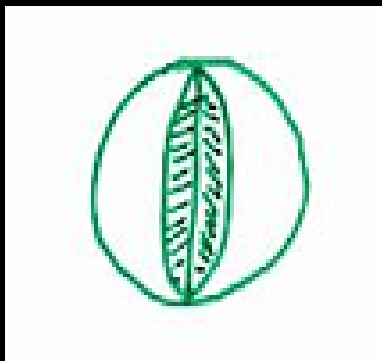
$$\text{If } T \rightarrow T + \delta T \text{ then } \hat{R} \rightarrow -\frac{\kappa^2 \rho}{\pi \alpha L^2} \quad \text{where } \rho = 2 \delta T$$

Setup

- Simple solution (SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-\phi_0} \quad \phi = \phi_0$$



Field equations

$$\frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2}$$

$$\kappa^2 Q^2 L^2 = 1 \quad \hat{R} = 0$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$

Obstructs T to δT

Setup

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$

Obstructs T to δT

- On other hand, general argument:

$$\rho = \int dV L_{bulk} = -\frac{1}{2\kappa^2} \int dV \partial^2 \phi = \oint dS n \cdot \partial \phi \propto \frac{\partial T}{\partial \phi}$$

Setup

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi) *F + \dots$$

Setup

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi) * F + \dots$$

- New function Φ has interpretation as brane-localized flux

$$\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b e^\phi$$

Calculation

- More general solutions

$$ds^2 = e^{2W} \hat{g}_{mn} dx^m dx^n + dr^2 + e^{2B} d\theta^2$$

$$F_{r\theta} = Qe^{B-4W} \quad \phi = \phi(r)$$

Calculation

- Perturb brane properties

$$T \rightarrow T + \delta T(\phi)$$

- To evade time-dependence add current

$$\Delta L_{bulk} = J\phi \quad \text{or} \quad \Delta L_{bulk} = J$$

- Find general solution to linearized equations

$$\kappa^2 J L^2 \ll 1$$

Calculation

- Sample solutions

$$\delta W = W_0 + W_1 \cos\left(\frac{r}{L}\right)$$

$$\delta\phi = \phi_0 + \phi_1 \ln\left(\frac{1 - \cos(r/L)}{\sin(r/L)}\right) - \kappa^2 J L^2 \ln\left[\sin\left(\frac{r}{L}\right)\right]$$

and so on

Calculation

- Brane-bulk boundary conditions:

$$(e^B \phi')_b = \frac{\kappa^2}{2\pi} \left(\frac{\partial L_b}{\partial \phi} \right)$$

$$(e^B W')_b = \frac{\kappa^2}{4\pi} \left(\frac{\partial L_b}{\partial g_{\theta\theta}} \right) = U_b$$

$$(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi} \left[\left(\frac{\partial L_b}{\partial \phi} + \frac{3}{2} \frac{\partial L_b}{\partial g_{\theta\theta}} \right) \right]$$

$$\text{Constraint: } 4U_b [2 - 2L_b - 3U_b] - \left(\frac{\partial L_b}{\partial \phi} \right)^2 = 0$$

Calculation

- Non-SUSY result:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[\frac{\pi\alpha L^2 \hat{R}(\phi)}{\kappa^2} \right]$$

$$\left[\frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*} = 0$$

$$\rho = \left[\sum_b \delta T_b - 2Q \delta \Phi_b \right]_{\phi_*}$$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2} \frac{\partial}{\partial\phi} \sum_b \delta T_b - Q\delta\Phi_b \right]_{\phi_*} = 0$$

ie Einstein frame potential: $V = U(\phi)e^{2\phi}$

Calculation

- SUSY result:

$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2} \frac{\partial}{\partial\phi} \sum_b \delta T_b - Q\delta\Phi_b \right]_{\phi_*} = 0$$

$$\rho = [\delta T_b - 2Q\delta\Phi_b] = \left[-\frac{1}{2} \frac{\partial}{\partial\phi} \sum_b \delta T_b - Q\delta\Phi_b \right]_{\phi_*}$$

Applications

- Three intriguing choices:

Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = C e^{-\phi}$ then $V(\phi) = A e^{2\phi}$

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Case 1: scale invariant:

if δT independent of ϕ and $\delta\Phi = C e^{-\phi}$ then $V(\phi) = A e^{2\phi}$

Case 2: exponentially large volume:

$\delta T_b = A + B (\phi + \nu)^2$ with $\nu \sim 50$ then $r = L e^{-\phi/2} \gg L$

Applications

- Three intriguing choices:

Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

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Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

Brane action independent of ϕ stable against brane loops

Bulk loops generate corrections of order $e^{2\phi} = (1/r)^4$

Conclusions

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- Branes and brane back-reaction can have important implications for low-energy theory
 - Little explored beyond codimension one
- For codimension two:
 - Explicit matching between source and bulk known
 - Potentially useful applications: *very light scalars; exponentially large dimensions; progress on cosmological constant; de Sitter constructions...*



Fin