General Lepton Mixing in Holographic Composite Higgs Models

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In collaboration with Claudia Hagedorn, based on 1106.4021 [hep-ph] and work in progress

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A Composite Higgs coming from some strongly coupled theory might resolve the hierarchy problem

Calculability hard. Recent breakthrough: the composite Higgs paradigm is closely related to (relatively weakly coupled) theories in extra dimensions

Connection particularly clear in Randall-Sundrum warped models thanks to the celebrated AdS/CFT duality

Holographic Composite Higgs models (HCHM) are the subset of Composite Higgs models that resemble in some aspects their 5D weakly coupled counterparts

Flavour hierarchies are nicely explained by large RG effects (geography in extra dimensions)

(H)CHM are among the most promising and well motivated extensions beyond the SM

HCHM are of this form

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix}$$

Elementary sector: SM particles but Higgs (and possibly RH top quark)

Composite sector: unspecified strongly coupled theory with unbroken global symmetry $G \supset G_{SM}$

Mixing sector: mass mixing between SM fermions and fermion bound states of the composite sector

The Higgs field is a bound state of the composite sector and EWSB, like in the SM, occurs by its condensation (in contrast to Technicolor)

It might or might not be a pseudo-Goldstone boson of a spontaneously broken global symmetry. In the former case we get Gauge-Higgs-Unification (GHU) models, in the latter one ordinary Randall-Sundrum models with scalar Higgs field.

In this talk I will briefly show how it is possible to have a Holographic Composite Higgs theory of leptons explaining the observed masses and mixing in the neutrino sector, with **no conflict** with Lepton Flavour Violation (LFV) processes in the charged lepton sector

Key ingredient: non-abelian discrete symmetry

- General model-independent analysis

- Analysis based on symmetry considerations only

The simplest non-abelian discrete groups, such as A_4 or S_4, naturally predict at leading order two large and a vanishing angle, in what is called the tribimaximal (TB) mixing

Recent results from the T2K and MINOS experiment reported indication of a non-vanishing θ_{13}

The size of θ_{13} is large enough that an explanation in terms of sub-leading effects in a mixing pattern where it is vanishing at leading order (such as TB mixing) is somehow disfavoured.

New lepton mixing patterns based on discrete non-abelian symmetries where a non-vanishing θ_{13} is obtained at leading order have recently be found

[Torop, Feruglio,Hagedorn, 1107.3486]

Key assumption: the neutrino and the charged lepton mass matrices are invariant under two distinct subgroups G_ν and G_e of a flavour group G_f

The lepton mixing arises from the different embeddings of G_ν and G_e inside G_f

This scenario is naturally realized in Composite Higgs Models (CHM), where the group G_f is broken to G_ν in the elementary sector and to G_e in the composite sector!

The flavour symmetry group is

$$G_f = X \times \mathbf{Z}_N$$

N>2, X is an unspecified non-abelian group with 2 key features

1) it admits irreducible three-dimensional representations

2) contains $\mathbf{Z}_2 \times \mathbf{Z}_2$ and $\mathbf{Z}_N$ as non-commuting subgroups

Additional $\mathbf{Z}_N$ required to have a natural explanation of the hierarchy of the charged lepton masses

The flavour symmetry breaking pattern is the following:

$$X \times \mathbf{Z}_N \to \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$$ Elementary sector

$$\mathbf{3} \quad \to \quad (1,-1) + (-1,1) + (-1,-1)$$

$$X \times \mathbf{Z}_N \to \mathbf{Z}_N^D$$ Composite sector

$$(\mathbf{3}, \omega_N^k) \to \omega_N^{k+n_1} + \omega_N^{k+n_2} + \omega_N^{k+n_3}$$

$$n_1 + n_2 + n_3 = 0 \bmod N \qquad \omega_N = e^{\frac{2i\pi}{N}}$$

In the basis where the generator G_N of $\mathbf{Z}_N \subset X$ is diagonal, the $\mathbf{Z}_2 \times \mathbf{Z}_2$ generators G_1 and G_2 cannot be put in diagonal form

$$G_1 = V G_1^{diag} V^\dagger, \qquad G_2 = V G_2^{diag} V^\dagger\,,$$

$$G_1^{diag} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad G_2^{diag} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

The unitary matrix V eventually gives lepton mixing

Consider Majorana neutrinos

The most general $\mathbf{Z}_2 \times \mathbf{Z}_2$ invariant Majorana mass term for two neutrino triplets ϕ_i^1 and ϕ_i^2 is

$$\phi_i^1 M_{ij} \phi_j^2 \qquad \text{where}$$

$$M_\nu = V \mathrm{diag}(m_1, m_2, m_3) V^t$$

The charged lepton mass matrix is diagonal in this basis

$$U_{PMNS} = V^*$$

HCHM with Majorana neutrinos

$$l_R^\alpha : \quad (\mathbf{1}, \omega_N^{n_1}),\ (\mathbf{1}, \omega_N^{n_2}),\ (\mathbf{1}, \omega_N^{n_3}),$$

$$l_L^\alpha,\ \nu_R^\alpha : \qquad (\mathbf{3}, 1) \quad \alpha = 1, 2, 3$$

$$\mathcal{L}_{el} = \bar{l}_L^\alpha i\hat{D} l_L^\alpha + \bar{l}_R^\alpha i\hat{D} l_R^\alpha + \bar{\nu}_R^\alpha i\hat{\partial}\nu_R^\alpha - \frac{1}{2}(\bar{\nu}_L^{c,\alpha} M_{\alpha\beta}\nu_R^\beta + h.c.)$$

$$\mathcal{L}_{mix} = \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lL}}}\bar{l}_L^\alpha \Psi_{l_L,R}^\alpha + \frac{\lambda_{l_R}^\alpha}{\Lambda^{\gamma_{lR}^\alpha}}\bar{l}_R^\alpha \Psi_{l_R,L}^\alpha + \frac{\lambda_{\nu_R}}{\Lambda^{\gamma_{\nu R}}}\bar{\nu}_R^\alpha \Psi_{\nu_R,L}^\alpha + h.c.$$

$$\Psi_{l_L,R}^\alpha : \qquad \Delta = \frac{5}{2} + \gamma_{lL}$$

$$\Psi_{l_R,L}^\alpha : \qquad \Delta = \frac{5}{2} + \gamma_{lR}$$

$$\Psi_{\nu_R,L}^\alpha : \qquad \Delta = \frac{5}{2} + \gamma_{\nu R}$$

Integrate out composite fermion operators

$$M_{l,\alpha\beta} \sim \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lR}}} \frac{\lambda_{l_R}^{\alpha}}{\Lambda^{\gamma_{lR}^{\alpha}}} \langle \bar{\Psi}_{l_L}^{\beta} \Psi_{l_R}^{\alpha} \rangle \sim c_\alpha v_H \lambda_{l_L} \lambda_{l_R}^{\alpha} \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma_{lR}^{\alpha}+\gamma_{lL}}$$

Charged leptons mostly elementary: $\gamma_{lL}, \gamma_{lR}^{\alpha} > 0$

ν_R^{α} strongly mixes with composite sector: $\gamma_{\nu R} < 0$

Integrate out heavy Majorana fermions

$$M_{\nu,\alpha\beta} \simeq \hat{c}_\alpha \hat{c}_\beta v_H^2 \lambda_{l_L}^2 \lambda_{\nu_R}^2 \left(\frac{\mu}{\Lambda}\right)^{2(\gamma_{\nu R}+\gamma_{lL})} \left(V M_D^{-1} V^t\right)_{\alpha\beta},$$

Additional symmetries or weak breaking of flavor symmetry in composite sector required

$$\hat{c}_\alpha \simeq \hat{c}$$

Mass matrix diagonalized by $\nu_L \to V^* \nu_L$ and $U_{PMNS} = V^*$

The composite flavour symmetry $\mathbf{Z}_N^D$ forbids tree-level flavour changing decays of charged leptons mediated by the exchange of heavy vector resonances.

Tree-level flavour changing charged interactions are suppressed by the heavy RH neutrinos

The main source of flavour violation arises from the elementary sector, since the kinetic terms of the SM fermions are constrained in general to be only $\mathbf{Z}_2 \times \mathbf{Z}_2$ invariant

$$\bar{l}_L i\hat{D} l_L \to \bar{l}_L (1 + Z_l) i\hat{D} l_L$$

$$Z_l = V Z_l^D V^t$$

Let us now look for explicit 5D realizations of this scenario for some specific choice of G_f

We take $N = 3$ and

$$X = \Delta(6n^2), \quad n = 2, 4, 8$$

$\Delta(6n^2)$ are non-abelian finite sub-groups of $SU(3)$ of order $6n^2$

$$\Delta(6n^2) \simeq (\mathbf{Z}_n \times \mathbf{Z}_n) \rtimes S_3$$

They are also subgroups of the modular group $PSL(2, \mathbf{Z})$

$$\Delta(24) \simeq S_4$$

and gives rise to the usual TB mixing matrix

$$\begin{aligned} \sin\theta_{13} &= 0 \\ \sin^2\theta_{12} &= \frac{1}{3} \qquad (S_4) \\ \sin^2\theta_{23} &= \frac{1}{2} \end{aligned}$$

Two inequivalent embeddings exist for $\Delta(96)$ and $\Delta(384)$:

$$\begin{aligned}
\sin^2\theta_{13} &\simeq 0.045 \\
\sin^2\theta_{12} &\simeq 0.349 \quad (\Delta(96)) \\
\sin^2\theta_{23} &\simeq 0.651 \quad (M1) \quad 0.349 \quad (M2)
\end{aligned}$$

$$\begin{aligned}
\sin^2\theta_{13} &\simeq 0.011 \\
\sin^2\theta_{12} &\simeq 0.337 \quad (\Delta(384)) \\
\sin^2\theta_{23} &\simeq 0.424 \quad (M3) \quad 0.576 \quad (M4)
\end{aligned}$$

[Torop, Feruglio,Hagedorn, 1107.3486]

Compare with recent global fits:

$$\begin{aligned}
\sin^2\theta_{13} &\simeq 0.021 \pm 0.007 \\
\sin^2\theta_{12} &\simeq 0.306^{+0.018}_{-0.015} \\
\sin^2\theta_{23} &\simeq 0.42^{+0.08}_{-0.03}
\end{aligned}$$

[Fogli et al., 1106.6018]

$$\begin{aligned}
\sin^2\theta_{13} &\simeq 0.013^{+0.007}_{-0.005} \\
\sin^2\theta_{12} &\simeq 0.312^{+0.017}_{-0.015} \\
\sin^2\theta_{23} &\simeq 0.52^{+0.06}_{-0.07}
\end{aligned}$$

[Schwetz, Tortola and Valle, 1108.1376]

5D GHU warped model

$$G_{\text{gauge}} = SO(5) \times U(1)_X$$

Extra factors to minimize number of terms

$$G_f = \Delta(6n^2) \times \mathbf{Z}_3 \times \mathbf{Z}_3' \times \mathbf{Z}_3''$$

$$G_{f,\text{UV}} = \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_3 \times \mathbf{Z}_3''$$

$$G_{f,\text{IR}} = \mathbf{Z}_3^D \times \mathbf{Z}_3'$$

$(\mathbf{3}, 1, \omega_3, \omega_3)$,

$$\xi_{l,\alpha} = \begin{pmatrix} \left[\tilde{L}_{1,\alpha L}(-+), \, L_{\alpha L}(++)\right] \\ \hat{\nu}_{\alpha L}(-+) \end{pmatrix},$$

$$\xi_{\nu,\alpha} = \nu_{\alpha L}(--)$$

$(\mathbf{1}, \omega_3^{n_\alpha}, \omega_3, \omega_3)$,

$$\xi_{e,\alpha} = \begin{pmatrix} x_{\alpha L}(+-) & \\ \tilde{\nu}_{\alpha L}(+-) & Z_{\alpha L}(+-) \\ e_{\alpha L}(--) & \\ \left[\tilde{L}_{2,\alpha L}(+-), \, \hat{L}_{\alpha L}(+-)\right] & \end{pmatrix}$$

$(\mathbf{3}, 1, \omega_3, 1)$,

We consider the most general $G_{\text{flavour,IR,UV}}$ invariant mass terms at the IR and UV brane and compute the lepton mass spectrum

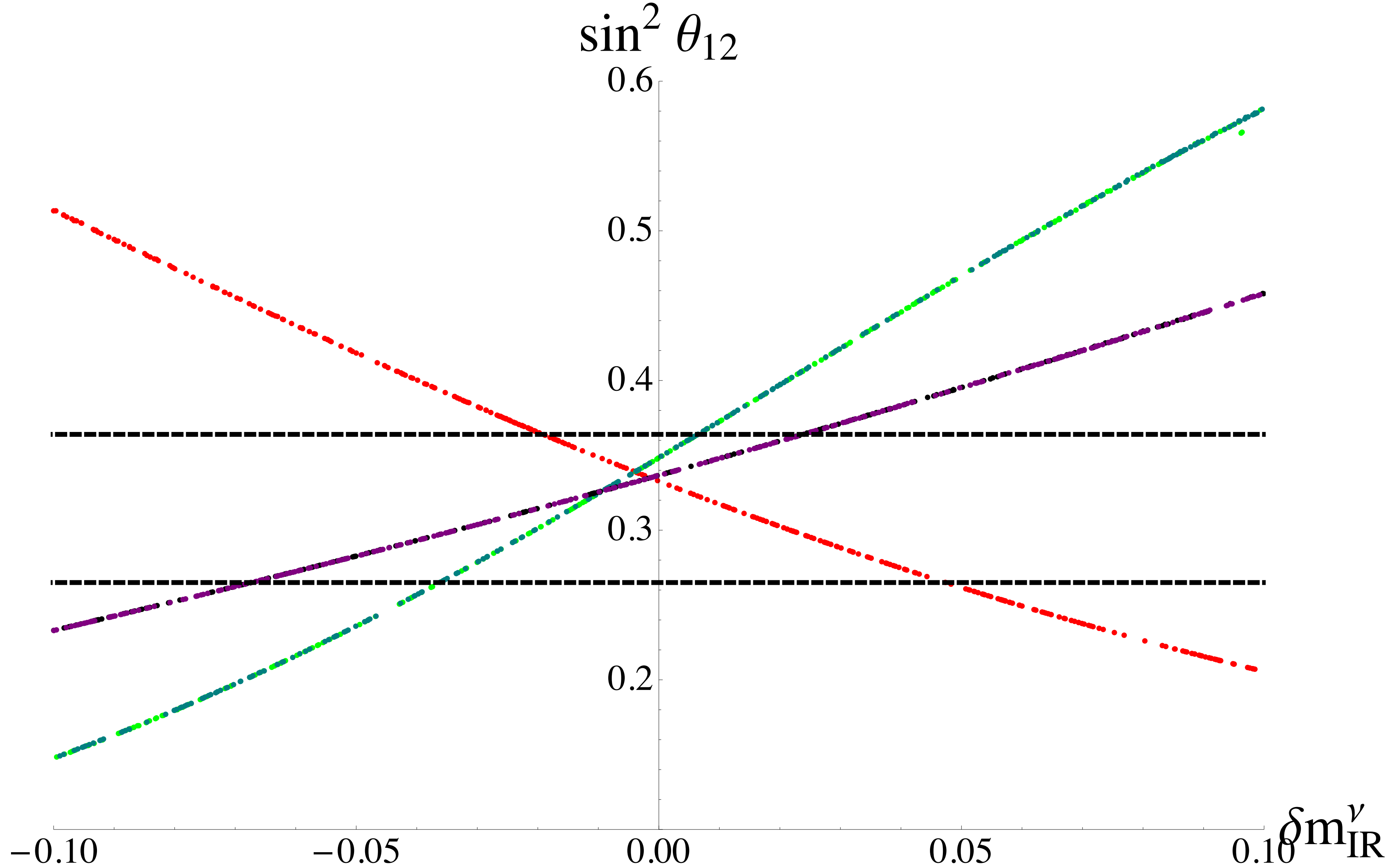
Charged lepton mass spectrum is hierarchical with no need of any flavour rotation

Lepton mixing requires that certain IR localized mass terms are close to universal. Otherwise they can all be set to one by means of an exchange $\mathbf{Z}_2$ symmetry

Resulting model is ultra-minimal with just one free parameter!

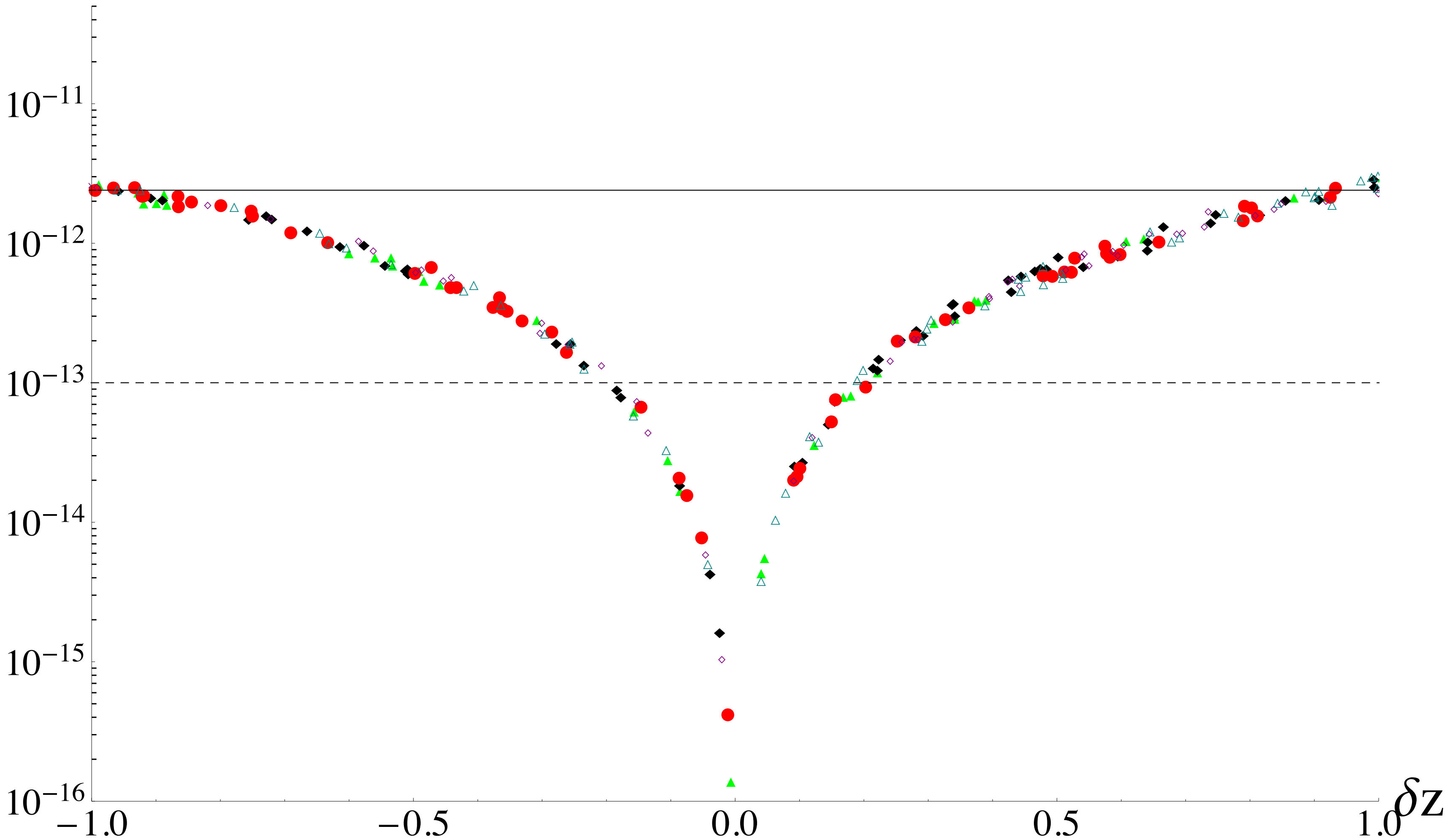
Leading LFV processes are induced by certain UV-localized fermion kinetic terms but are **negligible**

Normal hierarchy is preferred to inverted hierarchy



Red: S^4 Green: $\Delta(96)$ Purple: $\Delta(384)$

BR($\mu \to e\,\gamma$)



Red full circles S_4 Light green full triangles $\Delta(96)$, $M1$

Dark green empty triangles $\Delta(96)$, $M2$

Black full diamonds $\Delta(384)$, $M3$

Purple empty diamonds $\Delta(384)$, $M4$

Conclusions

We have proposed a simple class of scenarios based on the flavour symmetry $X \times \mathbf{Z}_N$ to accommodate leptons in HCHM

Lepton mixing with a non-vanishing θ_{13} angle can naturally be accommodated

All LFV processes are safely below the current experimental bounds

Back-up Slides

$\Delta(24)$ is defined by $\quad S^2 = T^4 = (ST)^3 = 1$

$$G_\nu : \qquad \{ST^2ST^2, T^2ST^2\}$$

$$G_e : \qquad (ST)^2$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} 0 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & -1 \\ -\sqrt{2} & -1 & 1 \end{pmatrix} \quad \rho(T) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & I \end{pmatrix}$$

$$V = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

$\Delta(96)$ is defined by $S^2 = (ST)^3 = T^8 = I, \quad (ST^{-1}ST)^3 = I$

$$G_\nu : \qquad \{S, ST^4ST^4\}$$

$$G_e : \qquad ST$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} e^{\frac{6\pi i}{4}} & 0 & 0 \\ 0 & e^{\frac{7\pi i}{4}} & 0 \\ 0 & 0 & e^{\frac{3\pi i}{4}} \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{2}(\sqrt{3}+1) & 1 & \frac{1}{2}(\sqrt{3}-1) \\ \frac{1}{2}(\sqrt{3}-1) & 1 & \frac{1}{2}(\sqrt{3}+1) \\ 1 & 1 & 1 \end{pmatrix}$$

$\Delta(384)$ is defined by $S^2 = (ST)^3 = T^{16} = I, \quad (ST^{-1}ST)^3 = I$

$$G_\nu : \qquad \{S, ST^8ST^8\}$$

$$G_e : \qquad ST$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix} \quad \rho(T) = \begin{pmatrix} e^{\frac{6\pi i}{8}} & 0 & 0 \\ 0 & e^{\frac{9\pi i}{8}} & 0 \\ 0 & 0 & e^{\frac{\pi i}{8}} \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{\sqrt{2}}{2}} & 1 & \sqrt{1+\frac{\sqrt{2}}{2}} \end{pmatrix}$$

Flavour invariant limit in composite sectors:

$$c_\alpha = 0, \quad \hat{c}_\alpha = \hat{c}$$

The flavour symmetry breaking in the composite sector for charged leptons (mainly tau) **cannot** be small because of universal gauge coupling deviations

$$\delta g_{l_{\alpha i}} \sim \frac{v_H^2}{m_\psi^2} (\lambda_{l_i}^\alpha)^2 \left(\frac{\mu}{\Lambda}\right)^{2\gamma_{li}^\alpha}, \quad i = L, R$$

$$\delta g_{l_{\alpha L}} \delta g_{l_{\alpha R}} \sim \left(\frac{M_{l,\alpha}}{m_\psi}\right)^2 \left(\frac{v_H}{m_\psi}\right)^2 \frac{1}{c_\alpha^2}$$

$$c_\alpha \lesssim 1 \text{ for } m_\psi \sim 1 \text{ TeV}$$

Integrate out composite fermion operators

$$M_{l,\alpha\beta} \sim \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lR}}} \frac{\lambda^{\alpha}_{l_R}}{\Lambda^{\gamma^{\alpha}_{lR}}} \langle \bar{\Psi}^{\beta}_{l_L} \Psi^{\alpha}_{l_R} \rangle \sim c_\alpha v_H \lambda_{l_L} \lambda^{\alpha}_{l_R} \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma^{\alpha}_{lR}+\gamma_{lL}}$$

Charged leptons mostly elementary: $\gamma_{lL}, \gamma^{\alpha}_{lR} > 0$

ν^{α}_{R} strongly mixes with composite sector: $\gamma_{\nu R} < 0$

Dominant kinetic term arises from strong coupling effects:

$$\delta_{\alpha,\beta} \tilde{c}^{2}_{\alpha} \lambda^{2}_{\nu_R} \left(\frac{\mu}{\Lambda}\right)^{2\gamma_{\nu R}} \int d^4x \, \bar{\nu}^{\alpha}_{R}(x) i \hat{\partial} \, \nu^{\alpha}_{R}(x)$$

Canonically normalized neutrino Dirac mass terms

$$M^{D}_{\nu,\alpha\beta} \sim \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lL}}} \frac{\lambda_{\nu_R}}{\Lambda^{\gamma_{\nu R}}} \left(\frac{\mu}{\Lambda}\right)^{-\gamma_{\nu R}} \frac{1}{\tilde{c}_\alpha \lambda^{\alpha}_{\nu_R}} \langle \bar{\Psi}^{\beta}_{l_L} \Psi^{\alpha}_{\nu_R} \rangle \sim \frac{\hat{c}_\alpha v_H \lambda_{l_L}}{\tilde{c}_\alpha} \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma_{lL}}$$