Studies of the two-Higgs-doublet model

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Scalars 2011

The two-Higgs-doublet model

- In the SM we have *one* Higgs doublet

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

- Gauge invariant and renormalizable Higgs potential

$$V_{\text{SM}} = -\mu(\varphi^\dagger \varphi) + \lambda(\varphi^\dagger \varphi)^2$$

- In the THDM the Higgs sector is extended

$$\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}$$

- THDM has five physical Higgs bosons: ρ', h', h'', $H^\pm$.
- Prominent example: Susy models like the MSSM

- THDM Higgs potential

H. E. Haber and R. Hempfling, PRD 48 (1993)

$$\begin{aligned} V = {} & m_{11}^2(\varphi_1^\dagger\varphi_1) + m_{22}^2(\varphi_2^\dagger\varphi_2) - \left[m_{12}^2(\varphi_1^\dagger\varphi_2) + h.c.\right] \\ & + \frac{\lambda_1}{2}(\varphi_1^\dagger\varphi_1)^2 + \frac{\lambda_2}{2}(\varphi_2^\dagger\varphi_2)^2 \\ & + \lambda_3(\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) + \lambda_4(\varphi_1^\dagger\varphi_2)(\varphi_2^\dagger\varphi_1) \\ & + \left[\frac{\lambda_5}{2}(\varphi_1^\dagger\varphi_2)^2 + \lambda_6(\varphi_1^\dagger\varphi_1)(\varphi_1^\dagger\varphi_2) + \lambda_7(\varphi_2^\dagger\varphi_2)(\varphi_1^\dagger\varphi_2) + h.c.\right], \end{aligned}$$

with m_{11}^2, m_{22}^2, $\lambda_{1,2,3,4}$ real and m_{12}^2, $\lambda_{5,6,7}$ complex.

Bilinears in the THDM

Bilinears

J. Velhinho, R. Santos and A. Barroso, **PLB 322** (1994),
O. Nachtmann, A. Manteuffel, MM **EPJC 48** (2006),
Nishi **PRD 74** (2006)

- General $SU(2)_L \times U(1)_Y$ gauge invariant terms of the potential for doublets:

$$\varphi_i^\dagger \varphi_j, \qquad (i,j=1,2).$$

- Arrange invariant scalar products into Hermitian 2×2 matrix

$$\underline{K} := \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 \end{pmatrix}.$$

- Decomposition by completeness of Pauli matrices and $\mathbb{1}_2$

$$\underline{K}_{ij} = \frac{1}{2}\left(K_0\, \delta_{ij} + K_a\, \sigma^a_{ij}\right).$$

- 4 real coefficients - bilinears - defined by this decomposition

$$K_0 = \varphi_i^\dagger \varphi_i, \qquad K_a = (\varphi_i^\dagger \varphi_j)\, \sigma_{ij}^a, \quad (a = 1, 2, 3).$$

- Inversion reads

$$\varphi_1^\dagger \varphi_1 = (K_0 + K_3)/2, \qquad \varphi_1^\dagger \varphi_2 = (K_1 + iK_2)/2,$$
$$\varphi_2^\dagger \varphi_2 = (K_0 - K_3)/2, \qquad \varphi_2^\dagger \varphi_1 = (K_1 - iK_2)/2.$$

- In terms of

$$K_0, \qquad \boldsymbol{K} \equiv \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix}$$

the most general potential can now be written

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- with real parameters

$$\xi_0,\ \eta_{00},\ \boldsymbol{\xi},\ \boldsymbol{\eta},\ E = E^{\mathrm{T}}$$

- Translation from conventional notation to bilinear space

$$\xi_0 = \frac{1}{2}(m_{11}^2 + m_{22}^2) \,, \quad \boldsymbol{\xi} = \frac{1}{2}\begin{pmatrix} -2\mathrm{Re}(m_{12}^2) \\ 2\mathrm{Im}(m_{12}^2) \\ m_{11}^2 - m_{22}^2 \end{pmatrix} ,$$

$$\eta_{00} = \frac{1}{8}(\lambda_1 + \lambda_2) + \frac{1}{4}\lambda_3 \,, \quad \boldsymbol{\eta} = \frac{1}{4}\begin{pmatrix} \mathrm{Re}(\lambda_6 + \lambda_7) \\ -\mathrm{Im}(\lambda_6 + \lambda_7) \\ \frac{1}{2}(\lambda_1 - \lambda_2) \end{pmatrix} ,$$

$$E = \frac{1}{4}\begin{pmatrix} \lambda_4 + \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6 - \lambda_7) \\ -\mathrm{Im}(\lambda_5) & \lambda_4 - \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6 - \lambda_7) \\ \mathrm{Re}(\lambda_6 - \lambda_7) & -\mathrm{Im}(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix} .$$

- We can even go ahead and write in an abstract *Minkowski space*

$$K = \begin{pmatrix} K_0 \\ K_1 \\ K_2 \\ K_3 \end{pmatrix} .$$

- The potential can thus be written in a very symmetric form with real parameters, ($\eta^{\mathrm{T}} = \eta$).

$$V = \xi_\alpha K_\alpha + \eta_{\alpha\beta} K_\alpha K_\beta , \qquad \alpha, \beta \in \{0, ..., 4\}$$

Example: maximally CP symmetric model

- We consider the THDM with the Higgs potential

$$
\begin{aligned}
V(\varphi_1, \varphi_2) = {} & m_{11}^2 \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) \\
& + \frac{1}{2} \lambda_1 \left((\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2 \right) \\
& + \lambda_3 (\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) + \lambda_4 (\varphi_1^\dagger \varphi_2)(\varphi_2^\dagger \varphi_1) \\
& + \frac{1}{2} \lambda_5 \left((\varphi_1^\dagger \varphi_2)^2 + (\varphi_2^\dagger \varphi_1)^2 \right) ,
\end{aligned}
$$

- Parameters m_{11}^2, λ_1, λ_3, λ_4, λ_5 are real.
- Potential invariant under $\varphi_1 \to -\varphi_1$.

- Translation to bilinears.

$$\xi_0 = m_{11}^2, \qquad \boldsymbol{\xi} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\eta_{00} = \frac{1}{4}(\lambda_1 + \lambda_3), \qquad \boldsymbol{\eta} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$E = \frac{1}{4} \begin{pmatrix} \lambda_4 + \lambda_5 & 0 & 0 \\ 0 & \lambda_4 - \lambda_5 & 0 \\ 0 & 0 & \lambda_1 - \lambda_3 \end{pmatrix}.$$

- Translation to bilinears.

$$\xi_0 = m_{11}^2, \qquad \boldsymbol{\xi} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\eta_{00} = \frac{1}{4}(\lambda_1 + \lambda_3), \qquad \boldsymbol{\eta} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$E = \frac{1}{4} \begin{pmatrix} \lambda_4 + \lambda_5 & 0 & 0 \\ 0 & \lambda_4 - \lambda_5 & 0 \\ 0 & 0 & \lambda_1 - \lambda_3 \end{pmatrix}.$$

- THDM potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- Translation to bilinears.

$$\xi_0 = m_{11}^2, \qquad \boldsymbol{\xi} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\eta_{00} = \frac{1}{4}(\lambda_1 + \lambda_3), \qquad \boldsymbol{\eta} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$E = \frac{1}{4} \begin{pmatrix} \lambda_4 + \lambda_5 & 0 & 0 \\ 0 & \lambda_4 - \lambda_5 & 0 \\ 0 & 0 & \lambda_1 - \lambda_3 \end{pmatrix}.$$

- THDM potential

$$V = \xi_0 K_0 + \cancel{\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K}} + \eta_{00} K_0{}^2 + \cancel{2 K_0 \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K}} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

Change of basis

- Consider the following mixing of the doublets

$$\begin{pmatrix} \varphi_1' \\ \varphi_2' \end{pmatrix} = U \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

with unitary 2×2 matrix U.

- The bilinears transform as

$$K_0' = K_0, \qquad K_a' = R_{ab}(U) K_b,$$

- where R is defined by

$$U^\dagger \sigma^a U = R_{ab}\, \sigma^b.$$

with matrix $R \in SO(3)$, that is proper rotations in $\boldsymbol{K}$-space.

- Under a change of basis $\boldsymbol{K} \to \boldsymbol{K}' = R\boldsymbol{K}$ the THDM potential remains invariant if we transform the parameters

$$\xi_0' = \xi_0, \quad \eta_{00}' = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

$$\begin{aligned} V =& \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{K} + \eta_{00} K_0{}^2 + 2K_0\,\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K} \\ =& \xi_0{}' K_0{}' + \boldsymbol{\xi}'^{\mathrm{T}} R R^{\mathrm{T}} \boldsymbol{K}' + \eta_{00}' K_0'^2 + 2K_0'\,\boldsymbol{\eta}'^{\mathrm{T}} R R^{\mathrm{T}} \boldsymbol{K}' + \boldsymbol{K}'^{\mathrm{T}} R R^{\mathrm{T}} E' R R^{\mathrm{T}} \boldsymbol{K}' \\ =& \xi_0' K_0' + \boldsymbol{\xi}'^{\mathrm{T}}\boldsymbol{K}' + \eta_{00}' K_0'^2 + 2K_0'\,\boldsymbol{\eta}'^{\mathrm{T}}\boldsymbol{K}' + \boldsymbol{K}'^{\mathrm{T}} E' \boldsymbol{K}', \end{aligned}$$

- That is we may diagonalize E by a change of basis and have 11 parameters of the THDM.

Symmetries

I. P. Ivanov **PRD 77** (2008), E. Ma, MM **PLB 683** (2010),
P. M. Ferreira, O. Nachtmann, J. P. Silva, MM **JHEP 1008** (2010),
P. M. Ferreira, H. E. Haber, O. Nachtmann, J. P. Silva, MM **IJMP A26** (2011)

- A transformation $K \to RK$, is a symmetry of the potential if and only if

$$\xi = R\,\xi, \qquad \eta = R\,\eta, \qquad E = R\,E\,R^{\mathrm{T}}.$$

CP transformations

Standard CP transformation

J.F. Gunion, H.E.Haber Phys.Rev.D72 (2005), I.F.Ginzburg, M.Krawczyk Phys.Rev.**D72** (2005), C.C. Nishi **PRD 74** (2006), O. Nachtmann, A. Manteuffel, MM **EPJ C57** (2008)

$$\varphi_i(x) \xrightarrow{\mathrm{CP_s}} \varphi_i^*(x'), \quad i = 1, 2, \quad x' = \begin{pmatrix} x_0 \\ -\boldsymbol{x} \end{pmatrix}$$

- In terms of bilinears

$$K_0(x) \xrightarrow{\mathrm{CP_s}} K_0(x'), \quad \begin{pmatrix} K_1(x) \\ K_2(x) \\ K_3(x) \end{pmatrix} \xrightarrow{\mathrm{CP_s}} \begin{pmatrix} K_1(x') \\ -K_2(x') \\ K_3(x') \end{pmatrix}.$$

- This is a reflection on the 1-3 plane

$$\boldsymbol{K}(\boldsymbol{x}) \xrightarrow{\mathrm{CP_s}} \bar{R}_2 \boldsymbol{K}(\boldsymbol{x}'), \quad \text{with } \bar{R}_2 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- With view on THDM Higgs potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} K_0{}^2 + 2 K_0 \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Potential is invariant under *standard* CP transformation if and only if there is a basis

$$\boldsymbol{\xi}' = R(U)\, \boldsymbol{\xi} = \begin{pmatrix} \cdot \\ 0 \\ \cdot \end{pmatrix}, \qquad \boldsymbol{\eta}' = R(U)\, \boldsymbol{\eta} = \begin{pmatrix} \cdot \\ 0 \\ \cdot \end{pmatrix},$$

$$E' = R(U)\, E\, R^{\mathrm{T}}(U) = \begin{pmatrix} \cdot & 0 & \cdot \\ 0 & \cdot & 0 \\ \cdot & 0 & \cdot \end{pmatrix}.$$

C. C. Nishi **PRD 74** (2006),
O. Nachtmann, A. Manteuffel, MM **EPJ C57** (2008)

- CP invariance conditions - basis invariant.

$$\xi^{\mathrm{T}} E\,(\xi \times \eta) = 0, \qquad (E\xi)^{\mathrm{T}} E\,(\xi \times (E\xi)) = 0,$$
$$\eta^{\mathrm{T}} E\,(\xi \times \eta) = 0, \qquad (E\eta)^{\mathrm{T}} E\,(\eta \times (E\eta)) = 0.$$

Potential is explicitly CP conserving if and only if these conditions are fulfilled.

- These conditions agree with former set of conditions, but are much simpler.

Generalized CP transformations

G.Ecker, W.Grimus, W.Konetschny, **NPB 191** (1981)

$$\varphi_i(x) \xrightarrow{\mathrm{CP_g}} U_{ij}\, \varphi_j^*(x'), \quad i,j = 1,2$$

- The bilinears transform as

O. Nachtmann, A. Manteuffel, MM **EPJC 57** (2007), O. Nachtmann, MM **JHEP 0905** (2009), P. Ferreira, J. Silva, **PR D83** (2011)

$$K_0(x) \xrightarrow{\mathrm{CP_g}} K_0(x'), \quad \boldsymbol{K}(x) \xrightarrow{\mathrm{CP_g}} \bar{R}\boldsymbol{K}(x')$$

with improper rotation $\bar{R}$.

- Requiring $\bar{R}^2 = \mathbb{1}_3$ there are two types

(i) $\bar{R} = -\mathbb{1}_3$, point reflection

(ii) $\bar{R} = R^{\mathrm{T}}\, \bar{R}_2\, R$, orthogonal equivalent to $\bar{R}_2$ reflection

Maximally CP-invariant model (MCPM)

- Potential invariant under point reflections

$$\boldsymbol{K}(x) \xrightarrow{\mathrm{CP}_g^{(i)}} -\boldsymbol{K}(x')$$

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} K_0{}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- that is we have to have

$$\boldsymbol{\xi} = \boldsymbol{\eta} = 0$$

- Note that this potential is automatically invariant under reflections on planes.

- Potential invariant under point reflections

$$\boldsymbol{K}(x) \xrightarrow{\mathrm{CP}_g^{(i)}} -\boldsymbol{K}(x')$$

$$V = \xi_0 K_0 + \cancel{\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K}} + \eta_{00} {K_0}^2 + \cancel{2 K_0 \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K}} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

- that is we have to have

$$\boldsymbol{\xi} = \boldsymbol{\eta} = 0$$

- Note that this potential is automatically invariant under reflections on planes.

Yukawa couplings in the MCPM

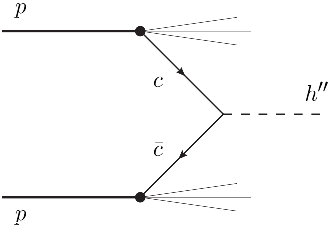
- At least two families for non-vanishing couplings.
- Absence of FCNC fixes couplings.
- Yukawa couplings

$$\mathscr{L}_{\text{Yuk},l}(x) = -c_{l\,3} \left\{ \bar{l}_{3\,R}(x)\, \varphi_1^\dagger(x) \begin{pmatrix} \nu_{3\,L}(x) \\ l_{3\,L}(x) \end{pmatrix} - \bar{l}_{2\,R}(x)\, \varphi_2^\dagger(x) \begin{pmatrix} \nu_{2\,L}(x) \\ l_{2\,L}(x) \end{pmatrix} \right\} + c.c.$$

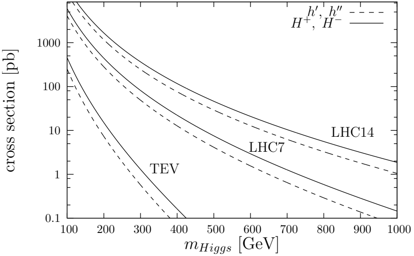
- Via EWSB $c_{l\,3}$ fixed, $m_{l_3} = c_{l\,3} \frac{\text{v}}{\sqrt{2}}$, v ≈ 246 GeV.
- Yukawa coupling of 2nd family prop. to 3rd family mass!

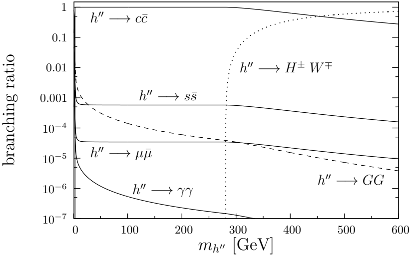
- Drell–Yan Higgs production dominant

O. Nachtmann, MM, JHEP 0905



- Very large cross section at LHC.

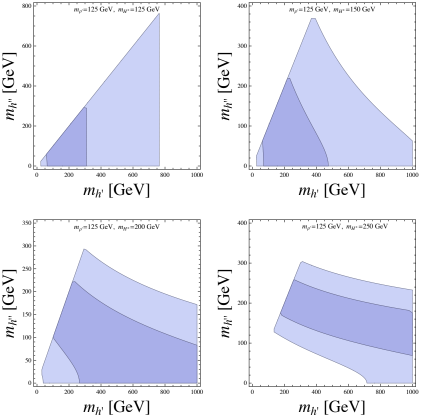




Oblique parameters

- Check agreement with electroweak measurements.
- Oblique parameters restrict viable parameter space.

O. Nachtmann, MM, arxiv:1106.1436 [hep-ph]



Conclusion

- Bilinears are quite powerful tool in general THDM potential.
- Basis-, CP transformations have simple geometric picture.
- Generalized CP transformations are point or plane reflections.
- Point-reflection invariance leads to a new *maximally CP-invariant model* (MCPM).
- Family replication in the MCPM.
- Phenomenology of the MCPM appealing.

- Tevatron luminosity 5 fb^{-1},
  LHC luminosity 100 fb^{-1}/year.
- Assuming Higgs boson masses h', h'', $H^{\pm}$ of 250 GeV

$$\sigma_{\text{Tevatron}} \approx 2 \text{ pb (10,000 events)},$$
$$\sigma_{\text{LHC}} \approx 1000 \text{ pb (100,000,000 events/year)}$$

- Decay proceeds mainly hadronically into c- and s-quarks.
- c-tagging maybe experimentally to difficult?
- Branching ratio $\frac{\Gamma(H \to \mu^- \mu^+)}{\Gamma(H \to \text{all})} \approx 3 \cdot 10^{-5}$ $(H = h', h'', H^{\pm})$.
- At Tevatron less than 1 event,
  at LHC we expect about 3000 events/year.

$SU(2)_L \times U(1)_Y$ breaking

- $SU(2)_L \times U(1)_Y$ breaking behavior in terms of K_0, K_1, K_2, K_3

$$\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}, \qquad \underline{K} := \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 \end{pmatrix}$$

- We have

$$\mathrm{Tr}\, \underline{K} = \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 = K_0 \geq 0$$

$$\det \underline{K} = (\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) - (\varphi_2^\dagger \varphi_1)(\varphi_1^\dagger \varphi_2) = K_0^2 - K_1^2 - K_2^2 - K_3^2 \geq 0$$

- K_0, $\boldsymbol{K}$ restricted to lie in *forward light cone*.

- Different domains with respect to EWSB. Consider minimum (vacuum) with

$K_0 = K_1 = K_2 = K_3 = 0$

$\varphi_1 = \varphi_2 = 0$
$SU(2)_L \times U(1)_Y$ **unbroken**

$K_0^2 > K_1^2 + K_2^2 + K_3^2$

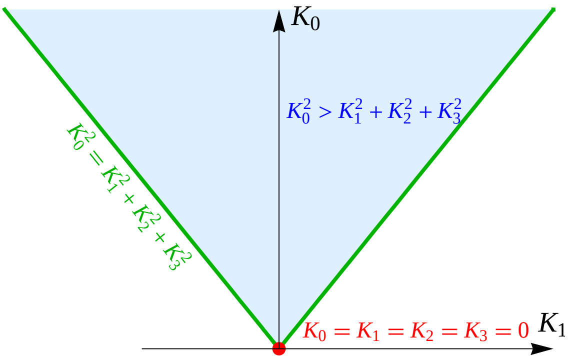
φ_1, φ_2 linear independent
Not possible to arrange $\varphi_1^+ = \varphi_2^+ = 0$
$SU(2)_L \times U(1)_Y$ **fully broken**

$K_0^2 = K_1^2 + K_2^2 + K_3^2$

φ_1, φ_2 linear dependent
Possible to arrange $\varphi_1^+ = \varphi_2^+ = 0$
$SU(2)_L \times U(1)_Y$ **partially broken.**

- Minkowski space structure of bilinears

O. Nachtmann, A. Manteuffel, MM **EPJC 48** (2006)



Spontaneous CP violation

- Potential invariant under CP transf. but vacuum not.
- Assume we have found a basis

$$\xi' = R(U)\,\xi = \begin{pmatrix} \cdot \\ 0 \\ \cdot \end{pmatrix}, \qquad \eta' = R(U)\,\eta = \begin{pmatrix} \cdot \\ 0 \\ \cdot \end{pmatrix},$$

$$E' = R(U)\,E\,R^{\mathrm{T}}(U) = \begin{pmatrix} \cdot & 0 & \cdot \\ 0 & \cdot & 0 \\ \cdot & 0 & \cdot \end{pmatrix}.$$

- Spontaneous CP violation is absent in this basis if and only if $\langle K_2 \rangle = 0$.
- Basis independent formulation:

$$(\boldsymbol{\xi} \times \boldsymbol{\eta})^{\mathrm{T}} \langle \boldsymbol{K} \rangle = 0, \qquad (\boldsymbol{\xi} \times (E\boldsymbol{\xi}))^{\mathrm{T}} \langle \boldsymbol{K} \rangle = 0,$$
$$(\boldsymbol{\eta} \times (E\boldsymbol{\eta}))^{\mathrm{T}} \langle \boldsymbol{K} \rangle = 0.$$

Translation of Higgs hypercharges

- In SUSY models the Higgs doublets (H_u and H_d) carry hypercharges $y = +1/2$ and $y = -1/2$.
- This can be translated to the convention used here by

$$\varphi_1^\alpha = -\epsilon_{\alpha\beta}(H_u^\beta)^*,$$
$$\varphi_2^\alpha = H_d^\alpha$$

with doublets

$$\varphi_i(x) = \begin{pmatrix} \varphi_i^+(x) \\ \varphi_i^0(x) \end{pmatrix} \qquad (i = 1, 2).$$

THDM invariant under point reflections

- In conventional notation we end up with

$$V(\varphi_1, \varphi_2) = m_{11}{}^2 \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2\right) + \frac{\lambda_1}{2} \left((\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2\right) + \lambda_3 (\varphi_1^\dagger \varphi_1)(\varphi_2^\dagger \varphi_2) + \lambda_4 (\varphi_1^\dagger \varphi_2)(\varphi_2^\dagger \varphi_1) + \frac{\lambda_5}{2} \left((\varphi_1^\dagger \varphi_2)^2 + (\varphi_2^\dagger \varphi_1)^2\right)$$

invariant under the four generalised $\mathrm{CP_g}$ transformations

$$\varphi_i(x) \xrightarrow{\mathrm{CP_g}} W_{ij}\ \varphi_j^*(x')$$

Unitary gauge

- In the unitary gauge we have

$$\varphi_1(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + \rho'(x) \end{pmatrix}, \quad \varphi_2(x) = \begin{pmatrix} H^+(x) \\ \frac{1}{\sqrt{2}}(h'(x) + ih''(x)) \end{pmatrix}$$

real fields: $\rho'(x),\ h'(x)$ and $h''(x)$
charged fields: $H^+(x),\ H^-(x) \equiv \left(H^+(x)\right)^*$

Yukawa coupling to one family

- Suppose, we couple one family of fermions to the Higgs doublets

$$\mathscr{L}_{\text{Yuk}}(x) = -\bar{l}_{1\,R}(x)\, c_{l\,i}\, \varphi_i^\dagger(x) \begin{pmatrix} \nu_{1\,L}(x) \\ l_{1\,L}(x) \end{pmatrix} + h.c.$$

  with $c_{l\,i}$ arbitrary complex numbers
- General ansatz for the $\text{CP}_g^{(i)}$ transformations of the fermions

$$\begin{pmatrix} \nu_{1\,L}(x) \\ l_{1\,L}(x) \end{pmatrix} \to e^{i\xi_1}\gamma^0\, S(C) \begin{pmatrix} \bar{\nu}_{1\,L}^{\text{T}}(x') \\ \bar{l}_{1\,L}^{\text{T}}(x') \end{pmatrix}$$

$$l_{1\,R}(x) \to e^{i\xi_2}\gamma^0\, S(C) \bar{l}_{1\,R}^{\text{T}}(x')\,,$$

  (γ^0 and $S(C) := i\gamma^2\gamma^0$ as usual)
- The Yukawa coupling is invariant under the $\text{CP}_g^{(i)}$ transformations only for

$$c_{l1} = c_{l2} = 0$$

Yukawa coupling to two families

- Suppose, we couple two families of fermions to the Higgs doublets

$$\mathscr{L}_{\text{Yuk}}(x) = -\bar{l}_{\alpha\,R}(x)\, C^{(j)}_{l\,\alpha\beta}\, \varphi_j^\dagger(x) \begin{pmatrix} \nu_{\beta\,L}(x) \\ l_{\beta\,L}(x) \end{pmatrix}, \quad \alpha, \beta = 2, 3$$

with $C_l^{(1)}$ and $C_l^{(2)}$ complex matrices.

- By field redefinitions one can always arrange that

$$C_l^{(1)} = \begin{pmatrix} c^{(1)}_{l\,2} & 0 \\ 0 & c^{(1)}_{l\,3} \end{pmatrix}, \quad c^{(1)}_{l\,2} \geq 0\,, \;\; c^{(1)}_{l\,3} \geq 0\,;$$

- Also the $\mathrm{CP_g}$ transformations may mix the families in this case

$$\begin{pmatrix} \nu_{\alpha\,L}(x) \\ l_{\alpha\,L}(x) \end{pmatrix} \to U^{(l)}_{L\,\alpha\beta}\, \gamma^0\, S(C)\, \left(\bar{\nu}^{\mathrm{T}}_{\beta\,L}(x'), \bar{l}^{\mathrm{T}}_{\beta\,L}(x')\right),$$

- The Yukawa coupling is now invariant only if

$$U_R^{(l)\,\mathrm{T}}\, C_l^{(j)\,*}\, U_L^{(l)\,*} W_{ji} = C_l^{(i)}\,. \tag{1}$$

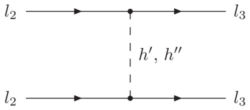
- Now we must find $U_L^{(l)}$ and $U_R^{(l)}$ for all different W.

We call a Lagrangian fullfilling (1) *maximal CP invariant*

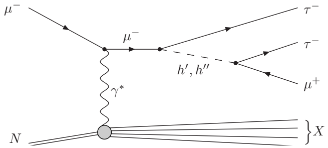
- Consider the case $c_{l\,2}^{(1)} > 0\,, \quad c_{l\,3}^{(1)} > 0\,, \quad c_{l\,2}^{(1)} \neq c_{l\,3}^{(1)}$
  - This corresponds to non-zero but different masses for l_2 and l_3.
  - The only possibility for $C_l^{(2)}$ is

$$C_l^{(2)} = \begin{pmatrix} 0 & c_{l\,23}^{(2)} \\ c_{l\,32}^{(2)} & 0 \end{pmatrix}$$

- This would lead to large FCNC's



- Example of muon–nucleon scattering process revealing FCNCs



- Consider the case $c_{l2}^{(1)} = c_{l3}^{(1)} > 0$
- This gives equal lepton masses, which is phenomenologically not acceptable.

- Consider the remaining case $c_{l2}^{(1)} = 0\,, \qquad c_{l3}^{(1)} > 0$
  - l_3 acquires a mass and l_2 is massless.
  - The only possibility for $C_l^{(2)}$ is now

$$C_l^{(2)} = \begin{pmatrix} -c_{l3}^{(1)} & 0 \\ 0 & 0 \end{pmatrix}$$

- We identify the two families with the II. and III. of the SM. The I. family is uncoupled.
- This seems to be justified to a certain extend.

	I			II			III	
u	2.4	MeV	**c**	1.27	GeV	**t**	172	GeV
d	4.8	MeV	**s**	105	MeV	**b**	4.2	GeV
e	0.511	MeV	μ	105.7	MeV	τ	1.777	GeV

Yukawa coupling Lagrangian

- We end up with the Yukawa coupling

$$\mathscr{L}_{\text{Yuk},l}(x) = -c^{(1)}_{l\,3} \left\{ \bar{l}_{3\,R}(x)\, \varphi_1^\dagger(x) \begin{pmatrix} \nu_{3\,L}(x) \\ l_{3\,L}(x) \end{pmatrix} - \bar{l}_{2\,R}(x)\, \varphi_2^\dagger(x) \begin{pmatrix} \nu_{2\,L}(x) \\ l_{2\,L}(x) \end{pmatrix} \right\} + h.c.$$

- After EWSB we get finally

$$\begin{aligned} \mathscr{L}_{\text{Yuk},l}(x) = & -m_{l\,3} \left(1 + \frac{\rho'(x)}{v_0} \right) \bar{l}_3(x)\, l_3(x) \\ & + \frac{m_{l\,3}}{v_0}\, h'(x)\, \bar{l}_2(x)\, l_2(x) + i \frac{m_{l\,3}}{v_0}\, h''(x)\, \bar{l}_2(x) \gamma_5 l_2(x) \\ & + \frac{\sqrt{2}\, m_{l\,3}}{v_0} \left[H^+(x)\, \bar{\nu}_2(x) \omega_R l_2(x) \; + H^-(x)\, \bar{l}_2(x) \omega_L \nu_2(x) \right] \end{aligned}$$

- Higgs–fermion couplings for II. family is prop. to $m_{l\,3}$
- The quark couplings are derived analogously.

Higgs decay

- Study of Higgs decay

$$H_1(k) \rightarrow f'(p_1) + \bar{f}(p_2)$$

- Decay rates can easily calculated from Lagrangian
- For the dominant contributions

$$h' \rightarrow c\bar{c}, \quad h'' \rightarrow c\bar{c}, \quad H^+ \rightarrow c\bar{s}, \quad H^- \rightarrow s\bar{c}$$

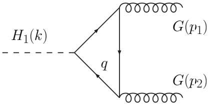
  we find rates of $\Gamma \approx 12$ GeV for $m_{H_1} = 200$ GeV.
- Study of Higgs decays

$$H_1(k) \rightarrow H_2(p_1) + V(p_2)$$

- We find that this decay rates become relevant only for a very heavy Higgs boson.

- Decay of neutral Higgs bosons into a gluon pair

$$H_1(k) \to G(p_1) + G(p_2)$$



- Calculation yields, i.e. for h'

$$\Gamma(h' \to G + G) = \frac{\alpha_s^2 m_{h'}}{32\pi^3} \left| \frac{2 m_t m_c}{v_0 m_{h'}} I\left(\frac{4m_c^2}{m_{h'}^2}\right) + \frac{2 m_b m_s}{v_0 m_{h'}} I\left(\frac{4m_s^2}{m_{h'}^2}\right) \right|^2$$

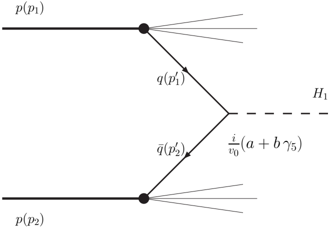
$$I(z) = \int_0^1 dv \frac{1-v}{z - v - i\epsilon} \ln\left(\frac{1+\sqrt{1-v}}{1-\sqrt{1-v}}\right)$$

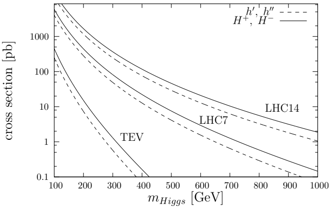
$$= 2 + (1-z) \begin{cases} -\frac{1}{2}\left[\ln\left(\frac{1+\sqrt{1-z}}{1-\sqrt{1-z}}\right) - i\pi\right]^2 & \text{for } 0 < z < 1 \\ 2\left[\arcsin(\sqrt{1/z})\right]^2 & \text{for } z \geq 1 \end{cases}$$

- This gives again tiny decay rates.

Higgs boson production in Drell–Yan

- Higgs production proceeds via





- Explicit calculation gives

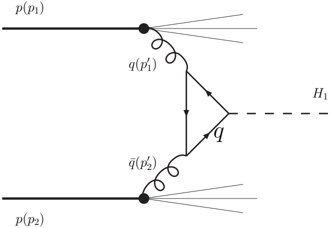
$$\sigma(p(p_1)+p(p_2)\to H_1(k)+X)\big|_{q\bar{q}-\text{fusion}} =$$

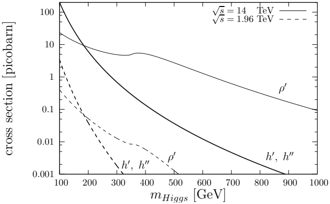
$$\frac{\pi}{3v_0^2 s}(|a|^2+|b|^2)\int_0^1 dx_1 N_q^P(x_1)\int_0^1 dx_2 N_{\bar{q}'}^P(x_2)\,\delta\Big(x_1x_2-\frac{m_{H_1}^2}{s}\Big)$$

- Cross section exceeding 100 pb for not to heavy Higgs m_{H_1}.

Neutral Higgs boson production via gluon fusion

- Higgs production $H_1 = \rho', h', h''$ proceeds via





- Explicit calculation gives

$$\sigma(p(p_1)+p(p_2)\to H_1+X)|_{GG-\text{fusion}} =$$

$$\frac{\pi^2\,\Gamma(H_1\to GG)}{8\ s\ m_{H_1}}\int_0^1 dx_1 N_G^p(x_1)\int_0^1 dx_2 N_G^p(x_2)\delta\Big(x_1x_2-\frac{m_{H_1}^2}{s}\Big)$$

Estimates of experimental detection of Higgs bosons

- At Tevatron we have data of 5 fb^{-1},
  at LHC we expect 100 fb$^{-1}/$year.
- Assuming a Higgs boson mass h', h'', $H^{\pm}$ of 250 GeV we find production cross sections of

$$\sigma_{\text{Tevatron}} \approx 2 \text{ pb, that is } \quad 10,000 \text{ events,}$$
$$\sigma_{\text{LHC}} \approx 1000 \text{ pb, that is } 100,000,000 \text{ events/year}$$

- Decay proceeds mainly hadronically into c- and s-quarks.
- c-tagging maybe experimentally to difficult?

Experimental Detection of Higgs bosons

- On the other hand we find branching ratios of

$$\frac{\Gamma(h' \to \mu^-\mu^+)}{\Gamma(h' \to \text{all})} \approx \frac{\Gamma(h'' \to \mu^-\mu^+)}{\Gamma(h'' \to \text{all})} \approx \frac{\Gamma(H^+ \to \mu^+\nu_\mu)}{\Gamma(H^+ \to \text{all})} \approx$$
$$\frac{\Gamma(H^- \to \mu^-\bar{\nu}_\mu)}{\Gamma(H^- \to \text{all})} \approx \frac{m_\tau^2}{3(m_t^2 + m_b^2) + m_\tau^2} \approx 3 \cdot 10^{-5} \ .$$

- Number of Higgs-bonsons with subsequent decay into μ:
  - At Tevatron less than 1 event.
  - At LHC we expect about 3000 events/year.

- Renormalization group equations for $\lambda_{1,2,3,4,5,6,7}$

$$
\begin{aligned}
8\pi^2 \frac{d\lambda_1}{dt} =& 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_6|^2 \\
& - \lambda_1 \left(\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right) + \frac{3}{8} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{8} g_2^4, \\
8\pi^2 \frac{d\lambda_2}{dt} =& 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_7|^2 \\
& - \lambda_2 \left(\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right) + \frac{3}{8} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{8} g_2^4, \\
8\pi^2 \frac{d\lambda_3}{dt} =& (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2|\lambda_6|^2 + 2|\lambda_7|^2 + 4\lambda_6\lambda_7^* + 4\lambda_6^*\lambda_7 \\
& - \lambda_3 \left(\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right) + \frac{3}{8} g_1^4 - \frac{3}{4} g_1^2 g_2^2 + \frac{9}{8} g_2^4, \\
8\pi^2 \frac{d\lambda_4}{dt} =& (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4|\lambda_5|^2 + 5|\lambda_6|^2 + 5|\lambda_7|^2 + \lambda_6\lambda_7^* + \lambda_6^*\lambda_7 \\
& - \lambda_4 \left(\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right) + \frac{3}{2} g_1^2 g_2^2, \\
8\pi^2 \frac{d\lambda_5}{dt} =& \lambda_5 \left(\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 \right) + 5\lambda_6^2 + 5\lambda_7^2 + 2\lambda_6\lambda_7 \\
& - \lambda_5 \left(\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right), \\
8\pi^2 \frac{d\lambda_6}{dt} =& 6\lambda_1\lambda_6 + 3\lambda_3(\lambda_6 + \lambda_7) + \lambda_4(4\lambda_6 + 2\lambda_7) + \lambda_5(5\lambda_6^* + \lambda_7^*)
\end{aligned}
$$

case	η_{01}	η_{02}	η_{03}	η_{12}	η_{13}	η_{23}	η_{11}	η_{22}	η_{33}	invariant terms
1)	0	0	$\surd$	$\surd$	0	0	$\surd$	$\surd$	$\surd$	$K_3,\ K_1K_2,\ K_1^2,\ K_2^2,\ K_3^2$
2)	$\surd$	η_{01}	0	$\surd$	$\surd$	$-\eta_{13}$	$\surd$	η_{11}	$\surd$	$K_1+K_2,\ K_1K_2,\ (K_1-K_2)K_3,$ $K_1^2+K_2^2,\ K_3^2$
3)	$\surd$	$-\eta_{01}$	0	$\surd$	$\surd$	η_{13}	$\surd$	η_{11}	$\surd$	$K_1-K_2,\ K_1K_2,$ $(K_1+K_2)K_3,\ K_3^2,$ $K_1^2+K_2^2$
4)	$\surd$	η_{01}	$-\eta_{01}$	$\surd$	$-\eta_{12}$	$-\eta_{12}$	$\surd$	η_{11}	η_{11}	$K_1+K_2-K_3,$ $K_1K_2-(K_1+K_2)K_3,$ $K_1^2+K_2^2+K_3^2$
5)	$\surd$	η_{01}	η_{01}	$\surd$	η_{12}	η_{12}	$\surd$	η_{11}	η_{11}	$K_1+K_2+K_3,$ $K_1K_2+K_1K_3+K_2K_3,$ $K_1^2+K_2^2+K_3^2$
6)	0	0	0	0	0	0	$\surd$	$\surd$	$\surd$	$K_1^2,\ K_2^2,\ K_3^2$
7)	0	0	0	$\surd$	0	0	$\surd$	η_{11}	$\surd$	$K_1K_2,\ K_1^2+K_2^2,\ K_3^2$
8)	0	0	0	$\surd$	$-\eta_{12}$	$-\eta_{12}$	$\surd$	η_{11}	η_{11}	$K_1K_2-(K_1+K_2)K_3,$ $K_1^2+K_2^2+K_3^2$