Vector bosons scattering and boundary conditions in KK toy model

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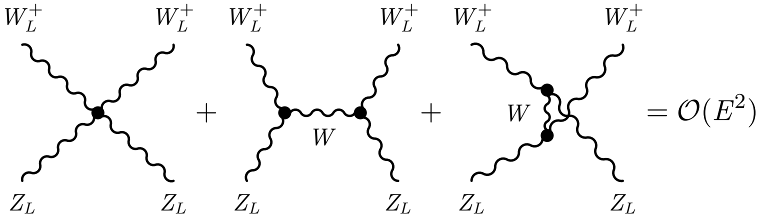
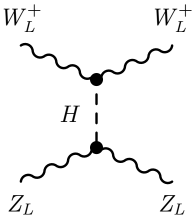
Standard Model

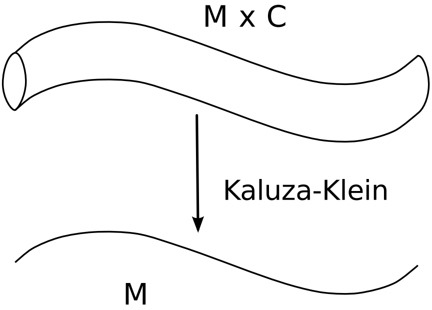
Higgs mechanism, tree-level unitarity





Higher-dimensional models
Compact extra dimension(s)

- Infinite space dimension compactified e.g. on S^1/Z_2 orbifold.



- Finite space dimension – “interval approach”.

Higher-dimensional models

Electroweak symmetry breaking

- Gauge Higgs Unification – Higgs field arises from extra component(s) of higher-dimensional gauge field.
- Higgsless models – Electroweak symmetry is broken via a non-trivial choice of boundary conditions for gauge field, no physical scalar is needed.

Pure gauge 5D theory on flat background

with one extra finite space dimension

- $y \in (0, \pi R)$
- $A_N^a(x^\mu, y) = (A_\nu^a(x^\mu, y), A_5^a(x^\mu, y))$
- $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5 f^{abc} A_M^b A_N^c$
- $\mathscr{L}_{\text{gauge}} = -\frac{1}{4} F_{MN}^a F^{aMN} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} F_{\mu 5}^a F^{a\mu 5}$
- R_ξ gauge: $\mathscr{L}_{\text{g.f.}} = -\frac{1}{2\xi} \left(\partial_\mu A^{a\mu} - \xi \partial_5 A_5^a\right)^2$

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Equations of motion, boundary conditions

Variational principle of least action ($\delta S = 0$) gives us

- equations of motion
  - $\partial^M F^a_{M\nu} - g_5 f^{abc} F^b_{M\nu} A^{cM} + \frac{1}{\xi}\partial_\nu \partial^\mu A^a_\mu - \partial^\nu \partial_5 A^a_5 = 0$
  - $\partial^\mu F^a_{\mu 5} - g_5 f^{abc} F^b_{\mu 5} A^{c\mu} + \partial_5 \partial^\mu A^a_\mu - \xi \partial_5 \partial_5 A^a_5 = 0$
- requirement of zero boundary terms
  - $\left[\left(\partial_\mu A^{a\mu} - \xi \partial_5 A^a_5\right) \delta A^a_5\right]_0^{\pi R} = 0$
  - $\left[F^{a\nu 5} \delta A^a_\nu\right]_0^{\pi R} = 0$

Possible boundary conditions

- $A^a_\mu\big|_{y=0,\pi R} = 0,\ \partial_5 A^a_5\big|_{y=0,\pi R} = 0$
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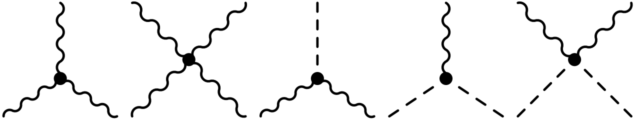
KK expansion, effective 4D Lagrangian

- KK expansion: $A_\mu^a(x,y) = \sum_n A_\mu^{a,n}(x)\,\varphi_{a,n}(y)$
- Boundary conditions keep the operator ∂_y^2 hermitian with respect to scalar product $\langle f, g\rangle = \int_0^{\pi R} f^*(y) g(y)\,\mathrm{d}y$
  $\Rightarrow$ complete orthonormal basis exists and satisfies
  $\varphi''_{a,n}(y) = -m_{a,n}^2 \varphi_{a,n}(y)$
- $\Box_5 A_\nu^a(x,y) - \partial_\nu \partial^\mu A_\mu^a(x,y) = 0 \Leftrightarrow$
  $(\Box_4 + m_{a,n}^2)\,A_\nu^{a,n}(x) - \partial_\nu \partial^\mu A_\mu^{a,n}(x) = 0$
- effective 4D Lagrangian: $\mathscr{L}_{\mathrm{4D}}(x) = \int_0^{\pi R} \mathscr{L}(x,y)\mathrm{d}y$

Interactions of effective 4D fields



- In unitary gauge ($\xi \to +\infty$) are all massive modes $A_5^{a,n}$ ($n > 0$) unphysical and play similar role as would-be Goldstone bosons of SM Higgs mechanism.

- Effective coupling constant in every vertex is given by an integral of wave functions, e.g. for 3V vertex:
  $g_{(a,i)(b,j)(c,k)} = g_5 \int_0^{\pi R} \varphi_{a,i}\, \varphi_{b,j}\, \varphi_{c,k}\, \mathrm{d}y$
- Scattering amplitudes of general $V_\mathsf{L}\, V_\mathsf{L} \to V_\mathsf{L}\, V_\mathsf{L}$ process do not grow explicitly with energy for any allowed choice of boundary conditions.

Example

Boundary conditions:

- $y = 0$: $A_\nu^{1,2} = 0$, $\partial_y A_\nu^3 = 0$
- $y = \pi R$: $\partial_y A_\nu^a = 0$

Wave functions and masses:

- $\varphi_{W,n}(y) = \sqrt{\frac{2}{\pi R}} \sin \frac{(2n-1)y}{2R} \Rightarrow m_{W,n} = \frac{(2n-1)}{2R}$, $n \geq 1$
- $\varphi_{Z,n}(y) = \sqrt{\frac{2}{2^{\delta_{k,0}} \pi R}} \cos \frac{2ny}{2R} \Rightarrow m_{Z,n} = \frac{2n}{2R}$, $n \geq 0$

Coupling constant:

- $g_{(W,i)(W,j)(Z,k)} = \frac{g_5}{\sqrt{2^{\delta_{k,0}} \pi R}} \left(\delta_{j,i+k} + \delta_{j,i-k} - \delta_{j,1-i+k}\right)$

Final remarks

- There are several ways how to break electroweak symmetry in higher-dimensional models.
- 5D theory is non-renormalizable, unitarity breakdown is postponed to the cutoff scale of effective 4D theory (related to the size of extra dimension).

Realistic models:

- More complicated gauge group, e.g. $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$.
- Warped extra dimensions.

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Thank you for your attention.

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arXiv:1106.3043 [hep-th]